AN EXPERIMENTAL INVESTIGATION OF
THE GENERAL INSTABILITY OF
RINGSTIFFEENED, UNPRESSURIZED,
THINWALLED CYLINDERS UNDER
AXIAL COMPRESSION

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AN EXPERIMENTAL INVESTIGATION OF THE GENERAL

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# ABSTRACT

Eight different series of thin-walled Mylar cylinders were tested experimentally to investigate the general instability of ring-stiffened, unpressurized, thin-walled cylinders under axial compressive loading. The primary objectives for these tests were: to determine whether the bending or the torsional stiffness of the rings was the most effective stiffening parameter; to determine the relative effectiveness of ring-stiffeners around the inside and the outside of the cylinders; and to investigate the mechanism of buckling of unpressurized cylinders under axial compression.

It was found that the torsional stiffness of the ring-stiffeners was the most important parameter for stiffening the thin-walled Mylar cylinders under axial loading. Ring-stiffeners with a low torsional stiffness did not stiffen the cylinder effectively until the rings were closely spaced. Ring-stiffeners on the inside of the cylinders did not affect the maximum buckling load when they were not bonded to the cylinder walls. Ring-stiffeners on the outside of the cylinders provided effective stiffening whether they were bonded to the cylinder walls or not.

The experimental results of thin-walled cylinders under axial compression indicated that the cylinder walls expand laterally to some critical amount, at which time they become unstable and suddenly collapse into buckling. The ring-stiffeners increase the critical compressive load with their effective torsional stiffness by resisting the annular collapse of the cylinder walls into diamond shaped buckles. At an L/R ring spacing ratio of 1.0 or less, the walls tended to buckle across the ring-stiffeners.

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L       length of the cylinder between support stiffeners

R       radius of the test cylinders (2.50 inches)

E       Mylar modulus of elasticity (780,000 psi)

$P_{cr}$       critical compression buckling load in pounds

$\sigma_{cr}$       critical compression buckling stress in the cylinder in psi

K       critical buckling coefficient for cylinders

t       actual cylinder Mylar wall thickness in inches

$t_1$       nominal thickness of the Mylar walls (0.0075 inches)

y       Poisson's ratio

# I

## INTRODUCTION

The objective of this study was to experimentally investigate
the general instability of ring-stiffened, unpressurized, thin-walled
cylinders subjected to axial compression. Ring-stiffened, thin-
walled cylinder construction is particularly appealing for use in the
design of aircraft and missiles where the weight-to-strength ratio
must be made as low as possible, consistent with other requirements.
Therefore, it is necessary to determine the controlling parameters
for optimum ring-stiffening of thin-walled cylinders. In Reference 1,
Shanley established a frame (or ring) coefficient parameter for
determining the effectiveness of cylinder ring-stiffeners. This
coefficient was a function of the ring bending stiffness. However, in
axial compression tests of ring-stiffened, unpressurized, thin-walled
aluminum cylinders conducted by Sechler (Ref. 2), the validity of this
ring-stiffener criteria was not substantiated. In these latter tests,
two cylinders were stiffened with rings having a L-shaped cross-
section which had a relatively large bending stiffness but a small
torsional stiffness. It was found that the low torsional stiffness had a
very detrimental effect on the effectiveness of the rings in preventing
buckling of the cylinder.

In the current study, two series of tests on ring-stiffened
cylinders in which the rings had the same torsional stiffness but differ-
ent bending stiffnesses were conducted to investigate the ring-stiffener
criteria. It was also desired to investigate the effect of ring spacing
and the effectiveness of relatively weak ring-stiffeners. Tests were

made to investigate the relative effectiveness of ring-stiffeners on the outside and on the inside of the cylinders.

Most of the previous work investigating the buckling of circular cylinders loaded under axial compression was noted and discussed in Reference 3.

The use of metal cylinders in experimental investigations has the disadvantages of high cost for the large sizes and difficulties of uniformity for the small sizes. Du Pont's plastic polyester film, Mylar, was investigated and used in the test cylinders in this study since the use of Mylar cylinders permitted a large number of cylinders to be tested relatively inexpensively. The tests were conducted at the California Institute of Technology.

# EQUIPMENT AND PROCEDURE

## A. Description of the Loading Apparatus

The loading apparatus shown in Figure 1 was used to axially compress the test cylinders. The essential parts consisted of the load ring, dial gage, and the fine pitch loading screw. The lower cap on the load ring was recessed to fit a 1/2 inch steel loading ball and the upper cap was drilled and threaded to take a bolt with a mounted steel loading ball of the same size. These two loading balls prevented bending or torsional moment loadings of the cylinders. The ball on the top of the load ring fitted into a recess in the loading screw. A ring was fitted on the bolt so that the mounted loading ball could not slip out of the loading screw recess, thereby permitting the load ring assembly to be left in place at all times. This so-called locking ring was attached to the loading screw by three screws to permit easy removal of the load ring assembly. The load ring was made from a 6 inch diameter section of "Shelby" pipe, 1/2 inch wide and 5/32 inch thick. A 5/16 inch diameter steel rod was attached to the inner surface of the top of the load ring and bent to the proper shape for the mounting for the dial gage. All of the mountings attached to the load ring were silver soldered.

The steel loading screw was 1 inch in diameter and 6 inches long. It was threaded with 40 threads per inch to provide fine increments of loading. The top of the loading screw was fitted with a brass disk that served as a handle for turning of the loading screw. This disk was silver soldered to the loading screw.

The frame for the apparatus consisted of two 15 inch square, 1/2 inch thick Dural mounting plates which were held in place by four 1/4 inch diameter, 32 inch long steel rods. These rods were threaded on each end and bolted to the plates. Strong, fine pitch threads for the loading screw were provided by a steel fitting which was joined to a second fitting through a hole in the center of the top mounting plate. Both fittings had large flanges to transmit the load to the plate.

The loading apparatus frame was mounted rigidly to insure uniform alignment of the loading and to prevent vibrating or shaking of the frame during testing. The effects of vibrations during cylinder compression testing were discussed in Reference 4.

The remainder of the equipment required in the experiments consisted of the end-plates for mounting of the cylinders. These consisted of two 3/4 inch thick, 6 inch diameter brass plates which were machined down to a 5 inch diameter but leaving a 1/4 inch thick flange. The flanges were essential for the initial alignment and the fine adjustment of the end-plates on the cylinders. The total weight of the top brass end-plate and the locking ring (5.8 pounds) was added to each compression test load. The top plate had a 1/8 inch hole drilled through it to allow the pressure inside the cylinders to equalize with the outside air pressure during the compression tests. Two split, steel bands rigidly clamped the cylinders on the end-plates. See Figure 2. The lugs were silver soldered to the split, steel rings, drilled and threaded to permit tightening.

B.  Description of the Test Cylinders

The test cylinders were made of du Pont's Mylar (polyester film) of 750 gage (7.5 mils nominal thickness), type A.  The Mylar came in sheets 36 inches by 36 inches and is described in Reference 5.  Although the tensile modulus was listed in a range of from 450,000 psi to 600,000 psi, tensile tests were conducted with several Mylar strips.  The Mylar had a linear stress-strain relationship up to a stress of approximately 7,000 psi and a tensile modulus of from 754,000 psi to 804,000 psi.  An average tensile modulus of 780,000 psi was used in the calculations in this report.  It was found that the 750A Mylar sheets varied in thickness.  A maximum of 8.3 mils and a minimum of 6.6 mils were measured in test specimens.

The Mylar sheets were cut into 16 inch length strips of varying widths which were wrapped around a 5 inch outside diameter brass tube and a seam made with a 3/16 inch wide strip of double-faced Scotchtape.  Then, 36 inch long bands of either 1/2 inch or 3/8 inch wide Mylar-Scotchtape were wrapped around the ends of the cylinder.  These end bands were necessary to prevent the steel locking bands from crimping the cylinder ends when mounting.

The cylinders were stiffened by support rings at specified intervals along the cylinder length.  The rings referred to hereafter as "thin Mylar-Scotchtape rings" were formed by wrapping 1/4 inch wide bands of Mylar-Scotchtape around the cylinder twice, resulting in a 0.0225 inch thick band.  The rings referred to hereafter as "thick Mylar-Scotchtape rings" were formed in the same manner as above

except that the bands were wrapped around the cylinder six times, resulting in a 0.0675 inch thick band. The remainder of the bands were made from 0.065 inch thick Plexiglass sheets. The rings referred to hereafter as "protruding-out Plexiglass rings" were precision machined into rings 0.065 inch thick, an outside diameter of 5.52 inches and an inside diameter of 5.02 inches. Thus, when mounted, the 1/4 inch dimension extended outward from the cylinder sides. The rings referred to hereafter as "protruding-in Plexiglass rings" were precision machined into rings 0.065 inch thick, an outside diameter of 5.00 inches and an inside diameter of 4.50 inches. In this case the 1/4 inch dimension extended inward from the cylinder inside wall. The rings referred to hereafter as "flat Plexiglass rings" were precision machined into strips 0.065 inch thick, 20 inches long and 1/4 inch wide. These strips were placed in an oven at a temperature of $200^{\circ}F$. The strips were first wrapped around a 3 inch diameter rod (which was also at $200^{\circ}F$) and held in place until relatively cool. They were then replaced in the oven to reheat, then wrapped around a 4 inch diameter disk (which was also at $200^{\circ}F$) and held in place until relatively cool. These split circular strips were ground to the desired circumference on a grinding wheel to provide suitable cylinder ring-stiffeners. When these rings were mounted on the cylinder, the 1/4 inch dimension extended in a direction parallel to the cylinder walls.

## C.  Mounting of the Test Cylinders

The cylinders were mounted by slipping the cylinders over the brass end-plates, not allowing the ends of the cylinder to touch the end-plate flanges.  (This last precaution precluded the possibility of non-uniform support of the cylinders.)  Then a 1/2 inch wide thin band of neoprene was wrapped around the ends of the cylinder over the previously mentioned Mylar-Scotchtape end bands and the steel locking rings placed over the neoprene bands and locked in place by tightening of the locking bolts.  It was found that better results were obtained when the split in the locking ring was bracketing the end of the Mylar-Scotchtape end band as there was less bunching up of this latter band.  The cylinder with the brass support plates locked in position was then placed on its side on a smooth hard surface.  A small square was used to check the perpendicularity of the end-plates with respect to the smooth surface.  If a misalignment was found, the appropriate locking ring was loosened and the plate pulled away from the cylinder to the correct position.  (It was found that pulling the plate resulted in better end support of the cylinder as pushing the plate towards the other for alignment often left a preloaded section of the cylinder which buckled prematurely when the cylinder compressive load was applied.)  Each end-plate was checked for perpendicularity through a complete revolution of the mounted cylinder.  Then the locking rings were firmly tightened and rechecked for any slippage.

## D. Mounting of the Support Rings

The rings were generally placed on a cylinder with the cylinder mounted on one of the brass end-plates. In the case of the thin and the thick Mylar-Scotchtape rings, it was found best to put the bands on the cylinder with the brass form still inside the cylinder. Better spacing and more uniform construction of the rings resulted. It was found that if these Mylar-Scotchtape rings were put on using too great a pull, the cylinder tended to assume an "hour-glass" shape which gave poor results. See Figures 3 and 4. Most of the cylinders with the thin Mylar-Scotchtape ring-stiffeners were 11 inches long or 10 inches between end support plates. With the closer ring spacing, this long cylinder took too long to construct and the results obtained by using a 6 inch (effectively 5 inches between end-plates) long cylinder were found to be consistent with the long cylinder test results. The same comments and results were obtained using the short cylinders with the thick Mylar-Scotchtape ring-stiffeners.

For the protruding-out Plexiglass rings, it was found easier to put the rings on with the brass form removed. Also, one Mylar-Scotchtape end support band had to be put on after the rings were mounted because of the precision fit of the rings on the cylinder. The rings were fixed to the cylinder by placing a 3/32 inch wide ring of Scotchtape on the cylinder first and then slipping the rings into place by slightly depressing the sides of the Mylar cylinder. It was found easiest and most precise to place the Scotchtape on used Mylar sheets, cut to the desired 3/32 inch width, peel the protective layer off the Scotchtape, wrap the narrow Mylar-Scotchtape band around the cylinder,

and then slowly peel off the Mylar strip. See Figure 5. By using thick bands of the desired width as spacers between rings, the ring distances could be made with a reasonable degree of accuracy.

In constructing the cylinders with the fixed, flat Plexiglass ring-stiffeners, first the Mylar strip was removed from the 1/4 inch wide Mylar-Scotchtape strips. The Scotchtape strips were placed on the inside of the rings, rather than on the cylinders as above. The protective layer for the Scotchtape was carefully removed and the rings put into place using the before-mentioned spacer bands. It was found necessary to bond the ends of these split, flat Plexiglass rings together after positioning to get the maximum ring support effect. General Electric household cement was found to be suitable for this purpose, although ethylene dibromide was used with some degree of success. The cement was faster, resulted in a reliable bond, and avoided the toxic problem of the ethylene dibromide. See Figure 6.

The so-called free, protruding-in Plexiglass rings were placed inside the cylinders and small Scotchtape tabs placed around the ring in four places to keep the rings in position. See Figure 7. The rings were spaced by visual reference to a spacer band around the outside of the cylinder.

The free, protruding-out Plexiglass ring-stiffened cylinders shown in Figure 8 were assembled in the same manner as were the fixed, protruding-out Plexiglass ring-stiffened cylinders except that no Scotchtape bonding strip was placed between the rings and the cylinder. Small tabs of Scotchtape were placed at four places around

the ring to prevent the ring from slipping from the desired ring spacing. Since the same ring-stiffeners were used for this unbonded series as in the bonded series, the cylinders were made slightly oversize in order to achieve the same degree of ring tightness as when these rings were bonded to the cylinder walls.

The most difficult of the Plexiglass ring-stiffener arrangements was the construction of the cylinders stiffened by the fixed protruding-in Plexiglass rings. See Figure 1. Again, the 3/32 inch wide strips of Mylar-Scotchtape were used for bonding, but in this case the Mylar backing strip was removed first rather than the protective layer on the Scotchtape. With the proper ring spacer in place on the outside of the cylinder, the Scotchtape protective layer strip was wrapped around the inside of the cylinder, taking care to prevent the ring spacer from slipping out of position. The protective layer was carefully removed to prevent the Scotchtape from being pulled loose. After cutting off the excess length of the Scotchtape strip, the Plexiglass ring was slipped into place. It was necessary to press against the cylinder walls to get the ring on the Scotchtape bond. It was also necessary to make the cylinders for these fixed, protruding-in Plexiglass rings slightly oversize to prevent the rings from bulging out the cylinder walls. The same amount of oversize was used as for the cylinders stiffened by the free, protruding-out Plexiglass rings.

E.  Test Procedure

A cylinder mounted on the brass end-plates was placed on the lower plate of the mounting frame.  A 1 inch diameter steel disk with a free, recessed steel ball was placed on top of the cylinder top plate and centered.  (A 1 inch diameter scribe mark had been made on the top plate to facilitate centering of this loading disk.)  The recessed cap on the bottom of the load ring assembly was slipped over the steel ball in the loading disk.  The loading screw was then screwed down until there was no clearance between the top screw mounted steel loading ball and the loading screw recess.  Then a small square was used to align the cylinder with the load ring.  The square was placed on top of the upper end-plate while the load ring was placed in two positions at right angles to each other.  The cylinder was shifted so that the load ring was perpendicular to the top end-plate of the cylinder.  Vertical loading of the cylinders was achieved by this procedure.

The load ring was then swung to the extreme position, touching the short rods extending upwards from the loading disk to prevent the ring from rotating as the loading screw was turned to apply the load. Molycoat was used to lubricate the steel loading balls and their recessed mountings, but it was insufficient lubrication for the upper steel loading ball as the ball wore out after approximately 100 test loadings.  Packing the loading screw recess with grease and putting Molycoat on the steel loading ball solved this problem.

The loading screw was turned at as uniform a rate as possible until buckling failure occurred.  The deflection of the cylinder when

failure occurred was obtained by maintaining a close watch on the dial
gage reading. (One division, 0.001 inch, could be read consistently.)
The corresponding failure load was obtained from the load ring cali-
bration curve. The stabilized deflection reading after buckling was
also recorded for information purposes to aid in determining how good
the loading had been. The load was then relieved and the cylinder
checked for any permanent deformations. In case there were no
permanent deformations of the Mylar, the cylinders could be either
reloaded to check the previous deflection loading or it could be rebuilt
with a different supporting ring spacing and tested again. Usually the
previous procedure was followed. It was more common, especially
for the closer ring spacing, for a permanent deformation to occur in
the Mylar during the first loading, thereby precluding any retesting.

## RESULTS AND DISCUSSION

The results of the axial compression tests on the cylinders
with the rigidly clamped ends are tabulated in Table I and shown
graphically in Figure 9. The axial compression loads were changed
into stresses from which the critical buckling coefficients were
determined from the following classical equations:

$$\sigma_{cr} = KE \left(\frac{t}{R}\right)$$

The previously mentioned tension tests of the Mylar determined a
yield point of approximately 7000 psi. A second series of tension tests
of the Mylar were conducted in the low stress range of from 30 psi to
1250 psi in which a vertical comparator was used to measure elonga-
tions. The results of both the high and the low stress tension tests
determined that the Mylar was well within its linear stress-strain
relationship in the cylinder compression tests where a maximum
buckling stress of 980 psi was determined. Hence, the corresponding
average modulus of elasticity, 780,000 psi, was used in these compu-
tations. A constant value of the R/t ratio of 333 was used, based upon
the nominal thickness of the Mylar.

An empirical equation from Reference 6 for the critical buckling
coefficient, K, in terms of the cylinder dimensions is as follows:

$$K = \frac{R}{t} \left[ 9\left(\frac{t}{R}\right)^{1.6} + 0.16 \left(\frac{t}{L}\right)^{1.3} \right]$$

This equation was plotted in Figure 9 to correlate the test results of the cylinders with the rigidly clamped ends with the results of previous investigations. It is apparent that the agreement is not exact. For the longest cylinders (10 inches) with rigidly clamped ends, the critical buckling coefficient given in Table I was found to be approximately 0.25, whereas the critical buckling coefficient determined from the above empirical equation was 0.28. Although these values are not the same, the difference is not great. A possible explanation of the differences between the empirically determined and the experimentally determined values of the critical buckling coefficients may lie in the difference in Poisson's ratios for the cylinder materials. This parameter does not appear in the above empirical equation but it is known that a decrease in Poisson's ratio tends to lower the value of K.

The results of the tests on the cylinders which were reinforced by the thin Mylar-Scotchtape rings are tabulated in Table II and presented in Figure 10. The curve for the cylinders with rigidly clamped ends determined from the plot in Figure 9 is redrawn in all the plots to provide a reference to show the ring-stiffening effectiveness. The results of the tests of the cylinders which were reinforced by the thick Mylar-Scotchtape rings are tabulated in Table III and also presented graphically in Figure 10. These plots show that the thicker rings provided a slight increase in cylinder strength at which buckling occurred when the rings were relatively widely spaced. The effectiveness of either the thin or thick Mylar-Scotchtape rings did not become appreciable until the ring spacing became quite close, and then the

effectiveness of the thick rings increased more rapidly as the spacing got even closer. The weaker thin Mylar-Scotchtape ring-stiffeners were relatively ineffective for all ring spacings investigated.

The test results of the cylinders stiffened by the fixed, protruding-out Plexiglass rings are tabulated in Table IV and plotted in Figure 11. It was noted that, at the wider ring spacing, the maximum buckling stresses of these cylinders were greater than that of the cylinders with their ends rigidly clamped. Several tests confirmed that this was a reproducible result. A comparison of the wall thicknesses of the cylinders involved in these tests indicate that they were essentially the same so that a possible explanation by a consideration of thickness effects was not possible.

The next series of cylinders were stiffened by fixed, flat, split Plexiglass rings. The results of the tests are tabulated in Table V and presented in Figure 12. The flat Plexiglass rings (which had the same torsional stiffness as the previous protruding-out Plexiglass rings) appeared to result in the same maximum buckling stresses of the cylinders at the wider ring spacing. As the ring spacing became closer, the maximum cylinder buckling stresses became less than those resulting from the protruding-out ring-stiffened cylinders. When the split ends of the rings were bonded together, the maximum cylinder buckling stresses again followed closely those of the protruding-out ring-stiffened cylinder. This result can be noted in Figure 12 by the test points of the fixed, flat, unbroken Plexiglass rings with respect to the reference curve of the fixed, protruding-out Plexiglass rings from Figure 11. A comparison of the test results of

the cylinders stiffened with the fixed, protruding-out Plexiglass rings and with the fixed, flat, unbroken Plexiglass rings, as shown in Figure 12, indicates that the torsional rigidity of the rings would be the criteria for maximum ring-stiffening effect. This hypothesis corroborates the test results of Sechler in Reference 2.

Due to the inward collapse of the cylinders into buckles, a series of test cylinders with the ring-stiffeners on the inside were conducted. In the first of this test series, the protruding-in Plexiglass rings were not bonded to the inside cylinder walls and hence are referred to as free. The rings were snug fitting but not so tight as to bow the cylinder walls outward. (In preliminary tests, a somewhat lower buckling stress resulted when the rings were fitted so tight as to cause the cylinder walls to bow out slightly.) The results of these tests are tabulated in Table VI and plotted in Figure 12. From the graph it is apparent that these ring-stiffened cylinders buckled at the same stress level as though there were no ring-stiffeners.

The next test sequence was conducted on cylinders stiffened by free or unbonded, protruding-out Plexiglass rings. The test results are tabulated in Table VII and presented graphically in Figure 13. The previously determined curves for the cylinders stiffened by fixed, protruding-out Plexiglass rings and by fixed, flat, split Plexiglass rings are drawn in to serve as references. From Figure 13, it is apparent that the unbonded, protruding-out Plexiglass rings resulted in a significant increase in the maximum cylinder buckling stresses, compared to the unstiffened cylinders.

The last series of axial compression tests on ring-stiffened
cylinders was with the protruding-in Plexiglass rings bonded (or
fixed) to the cylinder inner walls. The results of these tests are
listed in Table VIII and presented graphically in Figure 14. The
greater scatter of the test results of this test series was attributed
to three possible causes. The first explanation could be the thickness
effect. The wall thickness of the cylinders used in the cylinder test
for a L/R of 0.8 was 0.0082 inches, whereas the thickness of the
cylinder walls of almost all of the other L/R tests in this series was
0.0076 inches. In general, the cylinders with the thicker walls had
higher buckling stresses for the same L/R ratio. A thorough
investigation of this wall thickness effect was not attempted in the
experiments for this report. A second possible cause of the scatter
of the test results in this cylinder test series may be the effectiveness
of the Scotchtape bonding. Some non-uniformity of bonding effective-
ness of the Scotchtape should be expected which would have a greater
effect on the test results with the rings on the inside since the
unbonded inside rings were ineffective. A third consideration for
scatter of the bonded inside ring-stiffened cylinders was the relative
tightness of the rings in the cylinders. In preliminary tests, very
tight fitting, bonded inside rings resulted in very low cylinder buck-
ling stresses. These were made in a wide range of ring spacing and
low maximum buckling stresses resulted in each case. The test
results recorded and plotted in this report were from tests in which
the rings had varying degrees of snugness of fit. Due to the method
of cylinder construction, it was not possible to make fine adjustments

of the ring fit. The curves for the fixed, protruding-out and the
fixed, flat, split Plexiglass ring-stiffened cylinders were drawn in
for reference. It appeared that bonding the rings to the inside of the
cylinder did effectively increase the maximum buckling stresses,
whereas the previously mentioned unbonded inside rings had no
stiffening effect.

The ineffectiveness of the unbonded inside rings and the
relative effectiveness of the unbonded outside rings indicate that the
circumference of the cylinder loaded axially increases to some criti-
cal value where it cannot increase further without the cylinder
becoming unstable and buckling. The ring-stiffeners serve to increase
the critical load at which this instability occurs. It was determined
from the test results shown in Figure 11 that the torsional rigidity of
the ring-stiffeners was the primary factor in cylinder stiffening
effectiveness. It may be noted in Figure 12 that the critical buckling
stress for cylinders with the Plexiglass ring-stiffeners on the outside
increased linearly with decreasing ring spacing up to a spacing ratio
of approximately 0.7. It is believed that this linearly increasing
buckling strength occurs as a result of the rings preventing the early
collapse of the cylinder into large diamond-shaped buckles. As the
rings become closer, the permissible buckle size decreased and the
cylinder strength increased in proportion. With a ring spacing ratio
less than 0.7, the cylinders had a greater tendency to buckle across
the ring-stiffeners. This buckling tendency resulted in the walls of
the cylinder tending to rotate as well as translate into buckles under
the rings. The effectiveness of the ring torsional stiffness to stiffen
the cylinder increased sharply as the spacing ratio continued to decrease.

The tendency for the cylinders to buckle across the ring-stiffeners is apparent in Figures 2 and 3 where the Mylar-Scotchtape rings were not sufficiently rigid to prevent the cylinder from collapsing into large buckles which went across the rings. When the rings became close enough to provide a significant stiffening effect (a ring spacing ratio of less than 0.5), the cylinders failed as columns as indicated by their failure pattern. When the Plexiglass ring-stiffened cylinders with a close ring spacing buckled under axial compression, the cylinders tended to buckle across the rings as indicated by rings pulling away from the Scotchtape bonding strip after collapse. The cylinders buckled across the free, protruding-out Plexiglass rings and, in general, these cylinders buckled at a lower stress level than when these rings were bonded to the cylinder.

The above arguments indicate that the ring-stiffeners serve two purposes in providing effective stiffening. First, they restrain the lateral expansion of the cylinder walls during loading; and secondly, they restrain the rotational collapse of the cylinder walls into buckles. The increased stiffening effectiveness of bonding the inside rings to the cylinder tend to confirm these hypotheses.

It was discovered that, in constructing the Mylar-Scotchtape ring-stiffened cylinders, the rings could be put on so tight that the cylinders appeared to have an "hourglass" shape, i.e. bowed inward midway between end-plates. These cylinders would buckle at a much lower axial loading than if the rings were put on so that the cylinder sides were straight. Similar results were obtained in the cylinder tests with the rigidly clamped ends without ring-stiffeners. However,

if the cylinder walls were bowed slightly outward midway between end-plates, i.e. in a wooden barrel shape, a significant increase in the maximum buckling load was obtained. (In this type of test construction, the vertical seam of the cylinder had a tendency to slide apart.) The investigation of these bowed in or bowed out cylinders was primarily qualitative since control of the amount of bowing in or out was not attempted. It is apparent that a cylinder with the sides bowed in would have a tendency to bow in even more under axial loading. In this case, stiffening rings on the cylinder outside would be less effective than if the sides were straight or bowed out. This was confirmed by the experiments conducted in this study. Investigation of the effectiveness of ring-stiffeners on the cylinder inside when the walls are slightly bowed in was not attempted. It would appear they would provide more stiffening effect for the bowed in wall case than did the outside rings.

The cylinders with the rings placed on the inside, whether free or bonded, resulted in a significant increase in the post buckling load. The inside rings effectively reduced the size of the buckles and prevented permanent deformations in the Mylar cylinder walls after buckling.

The axial loading of the cylinders was applied by screwing down the loading screw by hand. The only consideration of the speed of load application was an attempt to turn the loading screw at as uniform a rate as possible. Although some variation of loading rate undoubtedly occurred, it is believed that the rate was sufficiently uniform so that any deviation of the critical buckling load was small. An

investigation of different loading rates was not considered pertinent
to the results in this study.

One of the most difficult conditions to control was the elimina-
tion of all vibrations of the test apparatus. As was noted previously in
Reference 4, any vibration of the cylinders during axial loading may
result in considerable scatter of the test results. The top and bottom
plates of the loading frame were fixed in position and did not introduce
any noticeable motions or vibrations. The primary source of vibra-
tions was the steel loading ball attached to the top of the load ring
which fitted into the recessed loading screw. The top loading ball and
the hemispherical recess into which it fit were coated with Molycoat
and the recess packed with grease. This reduced but never quite
eliminated the vibrations associated with the starting-friction each
time the loading screw was turned. The remaining vibrations were
not considered significant since the scatter of the test results was
small. An occasional cylinder test would have so little vibration
that an unusually high buckling load would result, but these loads were
not reproducible.

It was found that the 750A Mylar sheets varied considerably
from their nominal thickness of 0.0075 inches. The thinner sheets
provided the least consistent results and, in general, tended to give
lower buckling loads for the axially compressed, unpressurized cylin-
ders. Some sheets were found to vary as much as 15 per cent in
thickness, in which case they seldom gave reasonable or reproducible
test results.
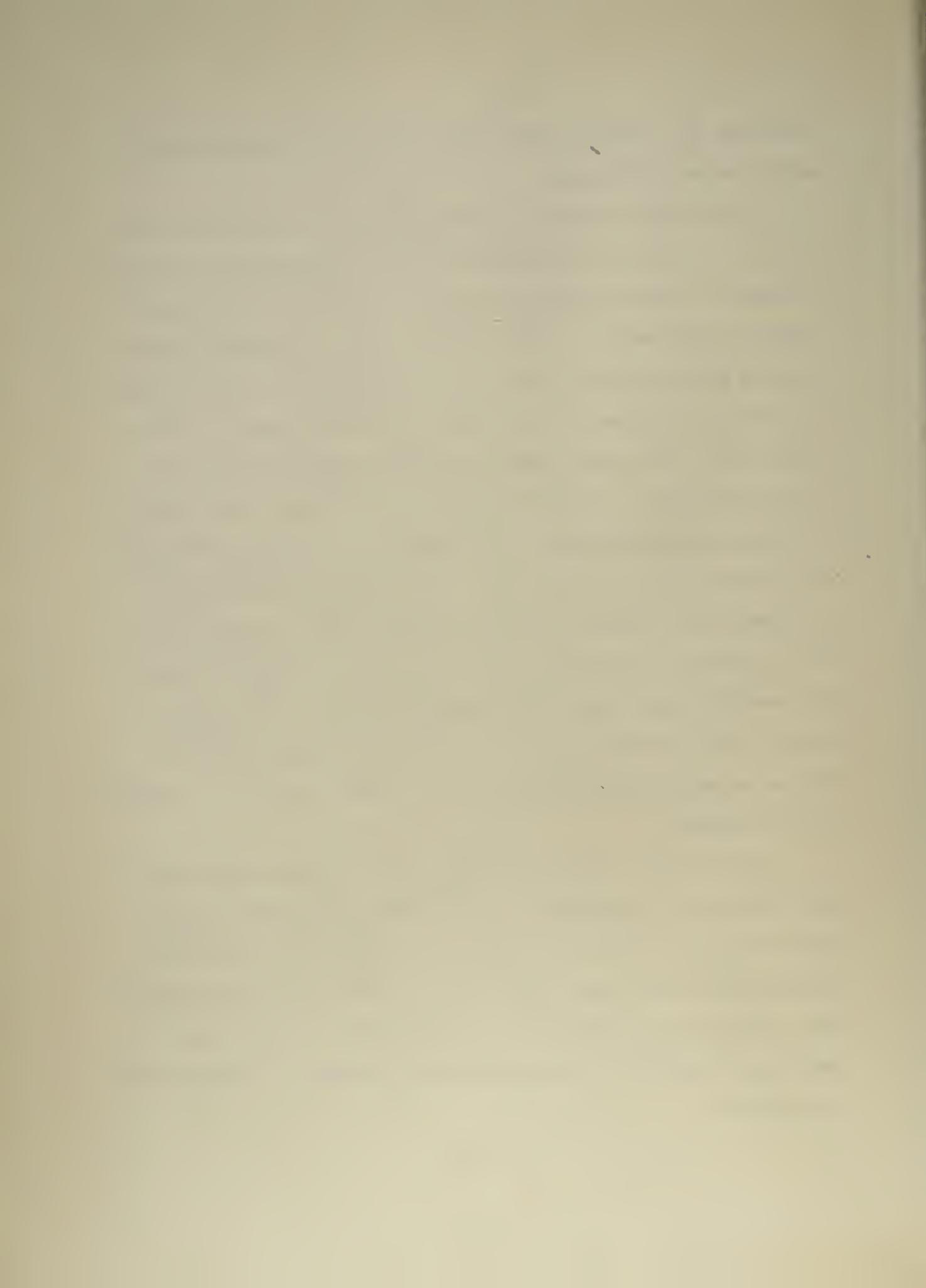
CONCLUSIONS

The following conclusions were reached, based on results of
the experiments conducted in this study:

1.  The torsional stiffness of ring-stiffeners on unpressurized,
thin-walled cylinders under axial compression was the important
parameter for optimum stiffening effectiveness.

2.  Torsionally weak ring-stiffeners were relatively ineffec-
tive even with the rings closely spaced.

3.  Ring-stiffeners placed on the inside of the cylinder under
axial compression had no stiffening effectiveness unless bonded to the
cylinder walls.

4.  Ring-stiffeners, free or bonded, placed on the inside of
the cylinder under axial compression effectively reduced the buckle
sizes after collapse, which resulted in a higher post buckling load.

5.  The critical buckling coefficient, K, obtained from the
classical equation for the critical buckling stress for cylinders

$$\sigma_{cr} = KE\left(\frac{t}{R}\right)$$

was found to be 0.25 for the unstiffened 10 inch long cylinders.

6.  The results of the experimental data indicate that when
unpressurized, thin-walled cylinders are subjected to axial compres-
sion the cylinder walls expand laterally to some critical amount, at
which time they become unstable and collapse suddenly into buckling.
Since the walls tend to buckle across the stiffeners when their L/R
spacing ratio is of the order of 1.0 or less, the local buckling of the

cylinder wall involves rotation as well as a lateral deflection. The ring-stiffeners increase the critical compressive load by the effectiveness of their torsional stiffness in resisting the collapse of the cylinder walls into buckles.

It is recommended that further experimental investigations be made to determine the effect of ring tightness on ring-stiffener effectiveness. This is especially important with regard to the ring-stiffeners on the inside of the cylinders. In connection with this, a determination of the effect of bonding strength of the rings to the cylinder walls should be made. It is further recommended that a serious program of study be carried out on the effectiveness of internal unbonded rings both for cylinders with and without additional longitudinal stiffeners.

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## TABLE I

## AXIAL COMPRESSION TESTS OF THIN MYLAR
## CYLINDERS WITH RIGID CLAMPED ENDS

Cylinder Radius R = 2.5 in., Mylar Nominal Thickness, $t_1$ = 0.0075 in.

| Test No. | Length, L (in.) | $\frac{L}{R}$ | Thickness, t (in.) | Load $P_{cr}$ (lb.) | Stress $\sigma_{cr}$ (psi) | $K^*$ |
|---|---|---|---|---|---|---|
| 1 | 10.0 | 4.00 | 0.0077 | 71.2 | 590 | 0.252 |
| 2 | 10.0 | 4.00 | 0.0075 | 68.8 | 585 | 0.250 |
| 3 | 10.0 | 4.00 | 0.0075 | 68.2 | 580 | 0.248 |
| 4 | 5.00 | 2.00 | 0.0074 | 69.3 | 596 | 0.255 |
| 5 | 5.00 | 2.00 | 0.0075 | 70.0 | 594 | 0.254 |
| 6 | 5.00 | 2.00 | 0.0075 | 70.0 | 594 | 0.254 |
| 7 | 4.75 | 1.90 | 0.0078 | 73.0 | 596 | 0.255 |
| 8 | 4.75 | 1.90 | 0.0078 | 74.8 | 610 | 0.261 |
| 9 | 3.25 | 1.30 | 0.0077 | 76.0 | 629 | 0.269 |
| 10 | 3.25 | 1.30 | 0.0078 | 77.1 | 630 | 0.269 |
| 11 | 3.25 | 1.30 | 0.0080 | 77.8 | 619 | 0.264 |
| 12 | 2.50 | 1.00 | 0.0078 | 80.2 | 654 | 0.280 |
| 13 | 2.50 | 1.00 | 0.0081 | 81.4 | 639 | 0.273 |
| 14 | 2.25 | 0.90 | 0.0080 | 82.0 | 653 | 0.279 |
| 15 | 2.20 | 0.88 | 0.0081 | 85.0 | 668 | 0.286 |
| 16 | 2.15 | 0.86 | 0.0083 | 86.8 | 675 | 0.289 |

$$^*K = (\frac{\sigma_{cr}}{E} \frac{R}{t_1})$$

## TABLE I (cont'd)

## AXIAL COMPRESSION TESTS OF THIN MYLAR
## CYLINDERS WITH RIGID CLAMPED ENDS

Cylinder Radius R = 2.5 in., Mylar Nominal Thickness, $t_1$ = 0.0075 in.

| Test No. | Length, L (in.) | $\frac{L}{R}$ | Thick-ness, t (in.) | Load, $P_{cr}$ (lb.) | Stress $\sigma_{cr}$ (psi) | $K^*$ |
|---|---|---|---|---|---|---|
| 17 | 2.00 | 0.80 | 0.0083 | 87.4 | 670 | 0.286 |
| 18 | 1.875 | 0.75 | 0.0083 | 89.2 | 685 | 0.293 |
| 19 | 1.75 | 0.70 | 0.0078 | 83.7 | 683 | 0.292 |
| 20 | 1.75 | 0.70 | 0.0080 | 86.2 | 686 | 0.294 |
| 21 | 1.75 | 0.70 | 0.0080 | 86.8 | 691 | 0.296 |
| 22 | 1.625 | 0.65 | 0.0078 | 83.1 | 678 | 0.290 |
| 23 | 1.25 | 0.50 | 0.0081 | 92.1 | 724 | 0.309 |
| 24 | 1.25 | 0.50 | 0.0081 | 93.9 | 738 | 0.316 |
| 25 | 1.25 | 0.50 | 0.0081 | 93.9 | 738 | 0.316 |
| 26 | 1.125 | 0.45 | 0.0079 | 94.5 | 761 | 0.326 |
| 27 | 0.875 | 0.35 | 0.0078 | 98.6 | 805 | 0.344 |
| 28 | 0.845 | 0.34 | 0.0075 | 99.1 | 841 | 0.360 |
| 29 | 0.75 | 0.30 | 0.0076 | 111.6 | 934 | 0.399 |
| 30 | 0.75 | 0.30 | 0.0074 | 107.0 | 921 | 0.394 |
| 31 | 0.75 | 0.30 | 0.0079 | 119.8 | 965 | 0.413 |
| 32 | 0.75 | 0.30 | 0.0078 | 116.9 | 953 | 0.407 |

$$^*K = (\frac{\sigma_{cr}}{E} \frac{R}{t_1})$$

## TABLE II

### AXIAL COMPRESSION TESTS OF THIN MYLAR CYLINDERS

### WITH THIN FLAT MYLAR-SCOTCHTAPE RINGS

Cylinder Radius R = 2.5 in., Mylar Nominal Thickness, $t_1$ = 0.0075 in.

| Test No. | Length, L (in.) | $\frac{L}{R}$ | Thick-ness, t (in.) | Load, $P_{cr}$ (lb.) | Stress $\sigma_{cr}$ (psi) | K[**] |
|---|---|---|---|---|---|---|
| 33 | 4.75 | 1.90 | 0.0075 | 69.3 | 586 | 0.251 |
| 34 | 4.75 | 1.90 | 0.0075 | 68.2 | 578 | 0.247 |
| 35 | 3.95 | 1.58 | 0.0076 | 70.6 | 592 | 0.253 |
| 36 | 3.95 | 1.58 | 0.0076 | 69.3 | 581 | 0.249 |
| 37 | 2.50 | 1.00 | 0.0075 | 70.6 | 599 | 0.256 |
| 38 | 2.50 | 1.00 | 0.0075 | 70.0 | 593 | 0.254 |
| 39 | 2.00 | 0.80 | 0.0075 | 69.3 | 586 | 0.251 |
| 40 | 2.00 | 0.80 | 0.0075 | 71.8 | 608 | 0.260 |
| 41 | 1.50 | 0.60 | 0.0074 | 70.0 | 602 | 0.258 |
| 42 | 1.50 | 0.60 | 0.0075 | 70.6 | 599 | 0.256 |
| 43 | 1.50 | 0.60 | 0.0075 | 71.8 | 608 | 0.260 |
| 44 | 1.50 | 0.60 | 0.0076 | 72.4 | 606 | 0.259 |
| 45 | 1.50 | 0.60 | 0.0076 | 72.4 | 606 | 0.259 |
| 46 | 1.00 | 0.40 | 0.0074 | 72.4 | 622 | 0.266 |
| 47 | 1.00 | 0.40 | 0.0074 | 70.6 | 608 | 0.260 |

[**]$K = (\frac{\sigma_{cr}}{E} \frac{R}{t_1})$

## TABLE II (cont'd)

## AXIAL COMPRESSION TESTS OF THIN MYLAR CYLINDERS

## WITH THIN FLAT MYLAR-SCOTCHTAPE RINGS

Cylinder Radius R = 2.5 in., Mylar Nominal Thickness, $t_1$ = 0.0075 in.

| Test No. | Length, L (in.) | $\frac{L}{R}$ | Thick- ness, t (in.) | Load, $P_{cr}$ (lb.) | Stress $\sigma_{cr}$ (psi) | K** |
|---|---|---|---|---|---|---|
| 48 | 1.00 | 0.40 | 0.0074 | 70.6 | 608 | 0.260 |
| 49 | 1.00 | 0.40 | 0.0075 | 71.2 | 605 | 0.259 |
| 50 | 1.00 | 0.40 | 0.0075 | 73.0 | 621 | 0.265 |
| 51* | 0.75 | 0.30 | 0.0082 | 82.0 | 637 | 0.272 |
| 52* | 0.75 | 0.30 | 0.0082 | 83.1 | 646 | 0.276 |
| 53* | 0.75 | 0.30 | 0.0082 | 83.7 | 651 | 0.278 |
| 54* | 0.75 | 0.30 | 0.0082 | 83.1 | 646 | 0.276 |
| 55* | 0.50 | 0.20 | 0.0078 | 88.5 | 722 | 0.309 |
| 56* | 0.50 | 0.20 | 0.0078 | 86.2 | 704 | 0.301 |
| 57* | 0.50 | 0.20 | 0.0078 | 86.8 | 708 | 0.303 |
| 58* | 0.50 | 0.20 | 0.0078 | 87.4 | 713 | 0.305 |
| 59* | 0.50 | 0.20 | 0.0081 | 91.5 | 719 | 0.307 |
| 60* | 0.50 | 0.20 | 0.0081 | 92.6 | 728 | 0.311 |
| 61* | 0.50 | 0.20 | 0.0081 | 92.1 | 724 | 0.309 |

*6 in. cylinders, all others 11 in. cylinders.

**$K = (\dfrac{\sigma_{cr}}{E} \dfrac{R}{t_1})$

## TABLE III

### AXIAL COMPRESSION TESTS OF THIN MYLAR CYLINDERS
### WITH THICK FLAT MYLAR-SCOTCHTAPE RINGS

Cylinder Radius R = 2.5 in., Mylar Nominal Thickness, $t_1$ = 0.0075 in.

| Test No. | Length, L (in.) | $\frac{L}{R}$ | Thick-ness, t (in.) | Load, $P_{cr}$ (lb.) | Stress $\sigma_{cr}$ (psi) | K[**] |
|---|---|---|---|---|---|---|
| 62[*] | 4.00 | 1.60 | 0.0080 | 74.8 | 596 | 0.255 |
| 63[*] | 4.00 | 1.60 | 0.0080 | 74.2 | 591 | 0.253 |
| 64[*] | 4.00 | 1.60 | 0.0080 | 74.8 | 596 | 0.255 |
| 65[*] | 2.50 | 1.00 | 0.0074 | 71.2 | 613 | 0.262 |
| 66 | 1.50 | 0.60 | 0.0077 | 74.8 | 619 | 0.264 |
| 67 | 1.50 | 0.60 | 0.0077 | 75.4 | 623 | 0.266 |
| 68 | 1.50 | 0.60 | 0.0077 | 74.8 | 619 | 0.264 |
| 69 | 1.50 | 0.60 | 0.0080 | 77.1 | 615 | 0.263 |
| 70 | 1.25 | 0.50 | 0.0079 | 77.8 | 627 | 0.268 |
| 71 | 1.00 | 0.40 | 0.0077 | 79.0 | 654 | 0.279 |
| 72 | 1.00 | 0.40 | 0.0077 | 80.8 | 669 | 0.286 |
| 73[*] | 1.00 | 0.40 | 0.0072 | 71.2 | 630 | 0.269 |

[*] 6 in. cylinders, all others 11 in. cylinders.

[**] $K = (\frac{\sigma_{cr}}{E} \frac{R}{t_1})$
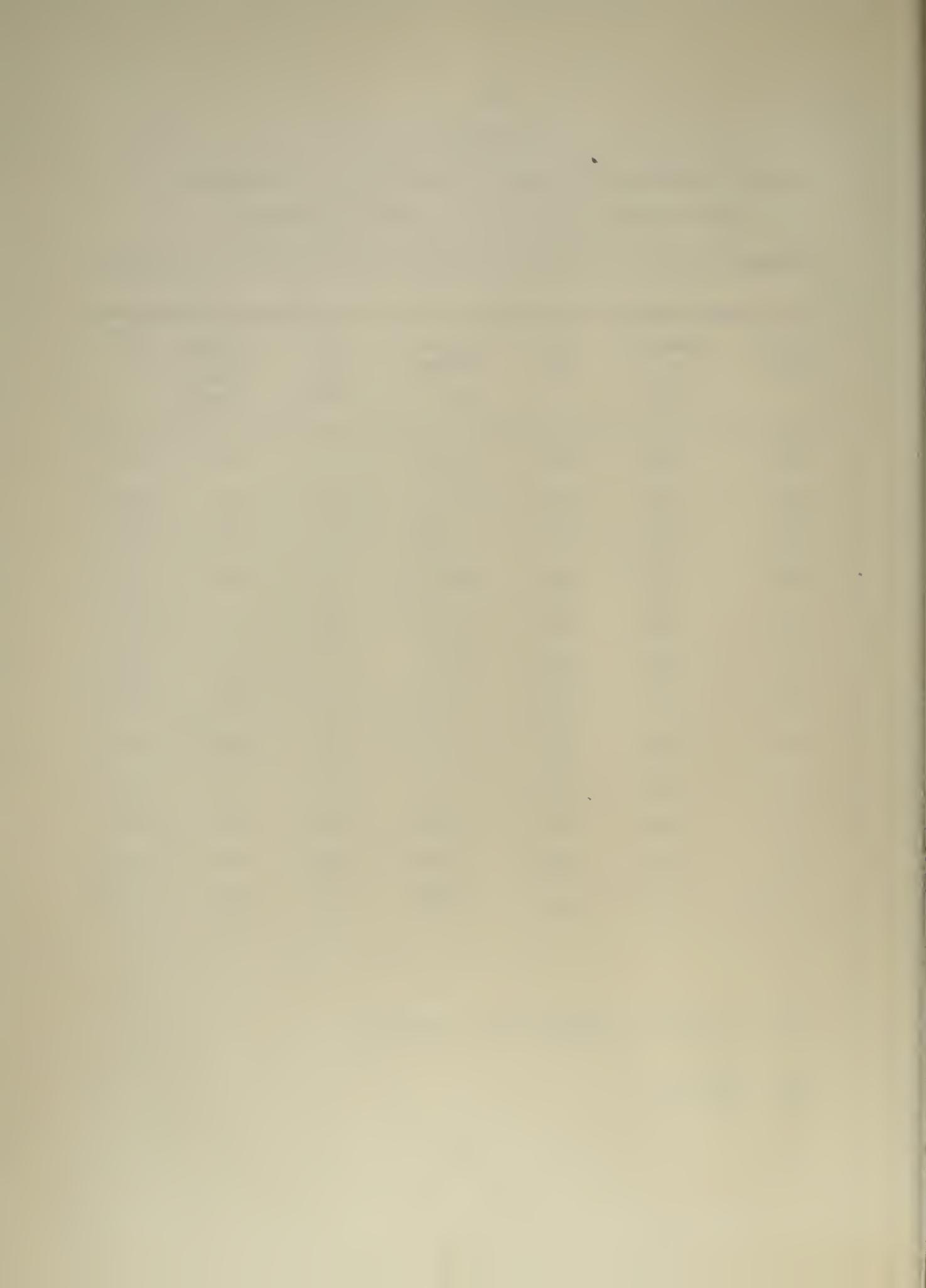
## TABLE III (cont'd)

## AXIAL COMPRESSION TESTS OF THIN MYLAR CYLINDERS

## WITH THICK FLAT MYLAR-SCOTCHTAPE RINGS

Cylinder Radius R = 2.5 in., Mylar Nominal Thickness, $t_1$ = 0.0075 in.

| Test No. | Length, L (in.) | $\frac{L}{R}$ | Thick- ness, t (in.) | Load, $P_{cr}$ (lb.) | Stress $\sigma_{cr}$ (psi) | K** |
|----------|-----------------|---------------|----------------------|----------------------|----------------------------|------|
| 74*      | 1.00            | 0.40          | 0.0075               | 82.0                 | 696                        | 0.298 |
| 75       | 0.875           | 0.35          | 0.0079               | 85.0                 | 685                        | 0.293 |
| 76       | 0.875           | 0.35          | 0.0079               | 84.4                 | 680                        | 0.291 |
| 77*      | 0.75            | 0.30          | 0.0075               | 90.9                 | 771                        | 0.330 |
| 78*      | 0.75            | 0.30          | 0.0074               | 88.5                 | 761                        | 0.326 |
| 79*      | 0.625           | 0.25          | 0.0077               | 99.1                 | 820                        | 0.351 |
| 80*      | 0.625           | 0.25          | 0.0077               | 98.0                 | 810                        | 0.346 |
| 81       | 0.625           | 0.25          | 0.0077               | 98.6                 | 815                        | 0.349 |
| 82       | 0.50            | 0.20          | 0.0077               | 104.0                | 860                        | 0.368 |
| 83       | 0.50            | 0.20          | 0.0077               | 102.8                | 850                        | 0.364 |
| 84       | 0.50            | 0.20          | 0.0076               | 107.6                | 900                        | 0.385 |

*6 in. cylinders, all others 11 in. cylinders.

**$K = (\dfrac{\sigma_{cr}}{E} \dfrac{R}{t_1})$

## TABLE IV

## AXIAL COMPRESSION TESTS OF THIN MYLAR CYLINDERS

## WITH FIXED PROTRUDING-OUT PLEXIGLASS RINGS

Cylinder Radius R = 2.5 in., Mylar Nominal Thickness, $t_1$ = 0.0075 in.

| Test No. | Length, L (in.) | $\frac{L}{R}$ | Thickness, t (in.) | Load, $P_{cr}$ (lb.) | Stress $\sigma_{cr}$ (psi) | $K^*$ |
|------|------|------|--------|------|-----|--------|
| 85 | 5.00 | 2.00 | 0.0076 | 73.0 | 611 | 0.261 |
| 86 | 5.00 | 2.00 | 0.0076 | 73.0 | 611 | 0.261 |
| 87 | 5.00 | 2.00 | 0.0076 | 71.8 | 601 | 0.257 |
| 88 | 5.00 | 2.00 | 0.0075 | 73.0 | 619 | 0.264 |
| 89 | 5.00 | 2.00 | 0.0075 | 71.2 | 603 | 0.258 |
| 90 | 4.00 | 1.60 | 0.0077 | 74.8 | 619 | 0.264 |
| 91 | 4.00 | 1.60 | 0.0078 | 76.6 | 625 | 0.267 |
| 92 | 3.00 | 1.20 | 0.0077 | 76.0 | 629 | 0.269 |
| 93 | 3.00 | 1.20 | 0.0077 | 76.6 | 634 | 0.271 |
| 94 | 3.00 | 1.20 | 0.0077 | 76.6 | 634 | 0.271 |
| 95 | 3.00 | 1.20 | 0.0077 | 76.6 | 634 | 0.271 |
| 96 | 3.00 | 1.20 | 0.0077 | 75.4 | 623 | 0.266 |
| 97 | 3.00 | 1.20 | 0.0077 | 74.8 | 619 | 0.264 |
| 98 | 2.50 | 1.00 | 0.0077 | 76.6 | 634 | 0.271 |

Cylinder Lengths: 5 in., 5 1/2 in., 6 in., 11 in.

$$^*K = (\frac{\sigma_{cr}}{E} \frac{R}{t_1})$$

## TABLE IV (cont'd)

## AXIAL COMPRESSION TESTS OF THIN MYLAR CYLINDERS

## WITH FIXED PROTRUDING-OUT PLEXIGLASS RINGS

Cylinder Radius R = 2.5 in., Mylar Nominal Thickness, $t_1$ = 0.0075 in.

| Test No. | Length, L (in.) | $\frac{L}{R}$ | Thick-ness, t (in.) | Load, $P_{cr}$ (lb.) | Stress $\sigma_{cr}$ (psi) | $K^*$ |
|---|---|---|---|---|---|---|
| 99 | 2.50 | 1.00 | 0.0077 | 77.8 | 643 | 0.275 |
| 100 | 2.50 | 1.00 | 0.0077 | 77.8 | 643 | 0.275 |
| 101 | 2.50 | 1.00 | 0.0077 | 77.8 | 643 | 0.275 |
| 102 | 2.50 | 1.00 | 0.0077 | 77.1 | 638 | 0.273 |
| 103 | 2.50 | 1.00 | 0.0077 | 78.4 | 649 | 0.277 |
| 104 | 2.50 | 1.00 | 0.0077 | 77.8 | 643 | 0.275 |
| 105 | 2.00 | 0.80 | 0.0076 | 78.4 | 656 | 0.280 |
| 106 | 2.00 | 0.80 | 0.0076 | 79.0 | 661 | 0.283 |
| 107 | 2.00 | 0.80 | 0.0076 | 77.8 | 651 | 0.278 |
| 108 | 2.00 | 0.80 | 0.0078 | 80.8 | 659 | 0.282 |
| 109 | 2.00 | 0.80 | 0.0078 | 79.5 | 649 | 0.277 |
| 110 | 2.00 | 0.80 | 0.0079 | 80.8 | 651 | 0.278 |
| 111 | 1.50 | 0.60 | 0.0077 | 79.5 | 657 | 0.281 |
| 112 | 1.50 | 0.60 | 0.0077 | 78.4 | 648 | 0.277 |

Cylinder Lengths: 5 in., 5 1/2 in., 6 in., 11 in.

$$^*K = (\frac{\sigma_{cr}}{E} \frac{R}{t_1})$$

## TABLE IV (cont'd)

## AXIAL COMPRESSION TESTS OF THIN MYLAR CYLINDERS

## WITH FIXED PROTRUDING-OUT PLEXIGLASS RINGS

Cylinder Radius R = 2.5 in., Mylar Nominal Thickness, $t_1$ = 0.0075 in.

| Test No. | Length, L (in.) | $\frac{L}{R}$ | Thick- ness, t (in.) | Load, $P_{cr}$ (lb.) | Stress $\sigma_{cr}$ (psi) | $K^*$ |
|---|---|---|---|---|---|---|
| 113 | 1.50 | 0.60 | 0.0077 | 80.2 | 663 | 0.283 |
| 114 | 1.50 | 0.60 | 0.0077 | 79.5 | 657 | 0.281 |
| 115 | 1.50 | 0.60 | 0.0080 | 85.6 | 682 | 0.292 |
| 116 | 1.50 | 0.60 | 0.0080 | 84.4 | 672 | 0.287 |
| 117 | 1.50 | 0.60 | 0.0079 | 83.1 | 670 | 0.286 |
| 118 | 1.50 | 0.60 | 0.0079 | 80.8 | 651 | 0.278 |
| 119 | 1.00 | 0.40 | 0.0077 | 83.1 | 687 | 0.294 |
| 120 | 1.00 | 0.40 | 0.0077 | 83.7 | 692 | 0.296 |
| 121 | 1.00 | 0.40 | 0.0077 | 88.5 | 732 | 0.313 |
| 122 | 1.00 | 0.40 | 0.0077 | 88.0 | 728 | 0.311 |
| 123 | 0.75 | 0.30 | 0.0075 | 92.7 | 786 | 0.336 |
| 124 | 0.75 | 0.30 | 0.0080 | 98.6 | 785 | 0.336 |
| 125 | 0.625 | 0.25 | 0.0078 | 108.1 | 882 | 0.377 |
| 126 | 0.50 | 0.20 | 0.0077 | 118.6 | 980 | 0.419 |

Cylinder Lengths:  5 in.,  5 1/2 in.,  6 in.,  11 in.

$^*K = (\dfrac{\sigma_{cr}}{E} \dfrac{R}{t_1})$

## TABLE V

## AXIAL COMPRESSION TESTS OF THIN MYLAR CYLINDERS

## WITH FIXED FLAT OUTSIDE PLEXIGLASS RINGS

Cylinder Radius R = 2.5 in., Mylar Nominal Thickness, $t_1$ = 0.0075 in.

| Test No. | Length, L (in.) | $\frac{L}{R}$ | Thickness, t (in.) | Load, $P_{cr}$ (lb.) | Stress $\sigma_{cr}$ (psi) | K[**] |
|---|---|---|---|---|---|---|
| 127 | 4.00 | 1.60 | 0.0072 | 70.0 | 619 | 0.264 |
| 128 | 3.00 | 1.20 | 0.0070 | 68.8 | 626 | 0.268 |
| 129 | 2.50 | 1.00 | 0.0077 | 78.4 | 648 | 0.277 |
| 130 | 2.00 | 0.80 | 0.0069 | 69.3 | 639 | 0.273 |
| 131 | 1.50 | 0.60 | 0.0074 | 79.0 | 679 | 0.290 |
| 132 | 1.00 | 0.40 | 0.0073 | 79.0 | 689 | 0.294 |
| 133 | 1.00 | 0.40 | 0.0072 | 76.6 | 678 | 0.290 |
| 134 | 0.875 | 0.35 | 0.0073 | 79.5 | 694 | 0.296 |
| 135 | 0.75 | 0.30 | 0.0077 | 87.4 | 723 | 0.309 |
| 136 | 0.75 | 0.30 | 0.0077 | 86.8 | 718 | 0.307 |
| 137 | 0.625 | 0.25 | 0.0070 | 83.1 | 756 | 0.323 |

Cylinder Length: 6 in.

[*]Unbroken rings, all others split

[**]$K = (\frac{\sigma_{cr}}{E} \frac{R}{t_1})$

## TABLE V (cont'd)

## AXIAL COMPRESSION TESTS OF THIN MYLAR CYLINDERS

## WITH FIXED FLAT OUTSIDE PLEXIGLASS RINGS

Cylinder Radius R = 2.5 in., Mylar Nominal Thickness, $t_1$ = 0.0075 in.

| Test No. | Length, L (in.) | $\frac{L}{R}$ | Thick-ness, t (in.) | Load, $P_{cr}$ (lb.) | Stress $\sigma_{cr}$ (psi) | $K^{**}$ |
|---|---|---|---|---|---|---|
| 138 | 0.50 | 0.20 | 0.0076 | 95.6 | 801 | 0.342 |
| 139* | 2.50 | 1.00 | 0.0068 | 65.1 | 610 | 0.261 |
| 140* | 1.50 | 0.60 | 0.0075 | 80.2 | 681 | 0.291 |
| 141* | 1.00 | 0.40 | 0.0073 | 82.6 | 721 | 0.308 |
| 142* | 1.00 | 0.40 | 0.0073 | 80.8 | 705 | 0.301 |
| 143* | 1.00 | 0.40 | 0.0070 | 80.2 | 730 | 0.312 |
| 144* | 1.00 | 0.40 | 0.0082 | 94.5 | 734 | 0.314 |
| 145* | 0.75 | 0.30 | 0.0076 | 97.4 | 815 | 0.348 |
| 146* | 0.625 | 0.25 | 0.0077 | 107.0 | 884 | 0.378 |
| 147* | 0.530 | 0.212 | 0.0074 | 107.6 | 924 | 0.395 |
| 148* | 0.50 | 0.20 | 0.0076 | 111.1 | 930 | 0.398 |

Cylinder Length: 6 in.

*Unbroken rings, all others split

$$**K = \left( \frac{\sigma_{cr}}{E} \frac{R}{t_1} \right)$$
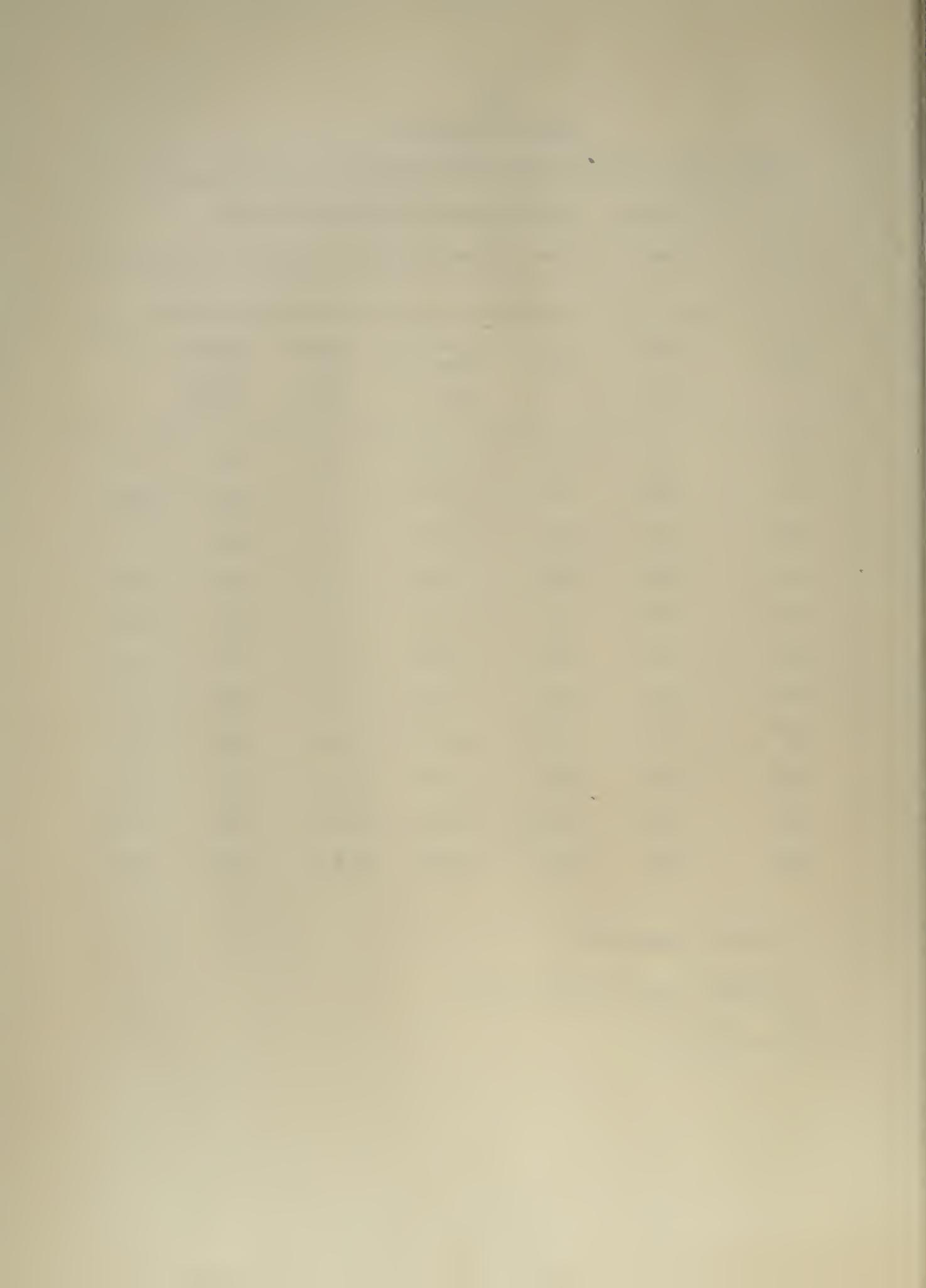
## TABLE VI

### AXIAL COMPRESSION TESTS OF THIN MYLAR CYLINDERS

### WITH FREE PROTRUDING-IN PLEXIGLASS RINGS

Cylinder Radius R = 2.5 in., Mylar Nominal Thickness, $t_1$ = 0.0075 in.

| Test No. | Length, L (in.) | $\frac{L}{R}$ | Thick- ness, t (in.) | Load, $P_{cr}$ (lb.) | Stress $\sigma_{cr}$ (psi) | K* |
|------|-------|------|--------|-------|------|-------|
| 149 | 5.00 | 2.00 | 0.0075 | 69.3 | 588 | 0.251 |
| 150 | 4.05 | 1.62 | 0.0080 | 72.4 | 576 | 0.246 |
| 151 | 4.05 | 1.62 | 0.0080 | 71.2 | 567 | 0.242 |
| 152 | 4.05 | 1.62 | 0.0080 | 71.8 | 571 | 0.244 |
| 153 | 2.00 | 0.80 | 0.0080 | 72.4 | 576 | 0.246 |
| 154 | 2.00 | 0.80 | 0.0080 | 72.4 | 576 | 0.246 |
| 155 | 1.25 | 0.50 | 0.0080 | 73.6 | 586 | 0.250 |
| 156 | 1.25 | 0.50 | 0.0080 | 74.2 | 591 | 0.253 |
| 157 | 1.00 | 0.40 | 0.0080 | 73.6 | 586 | 0.251 |
| 158 | 1.00 | 0.40 | 0.0080 | 74.2 | 591 | 0.253 |
| 159 | 1.00 | 0.40 | 0.0080 | 74.8 | 596 | 0.255 |
| 160 | 0.75 | 0.30 | 0.0074 | 68.8 | 592 | 0.253 |

Cylinder Length: 6 in.

*$K = (\frac{\sigma_{cr}}{E} \frac{R}{t_1})$
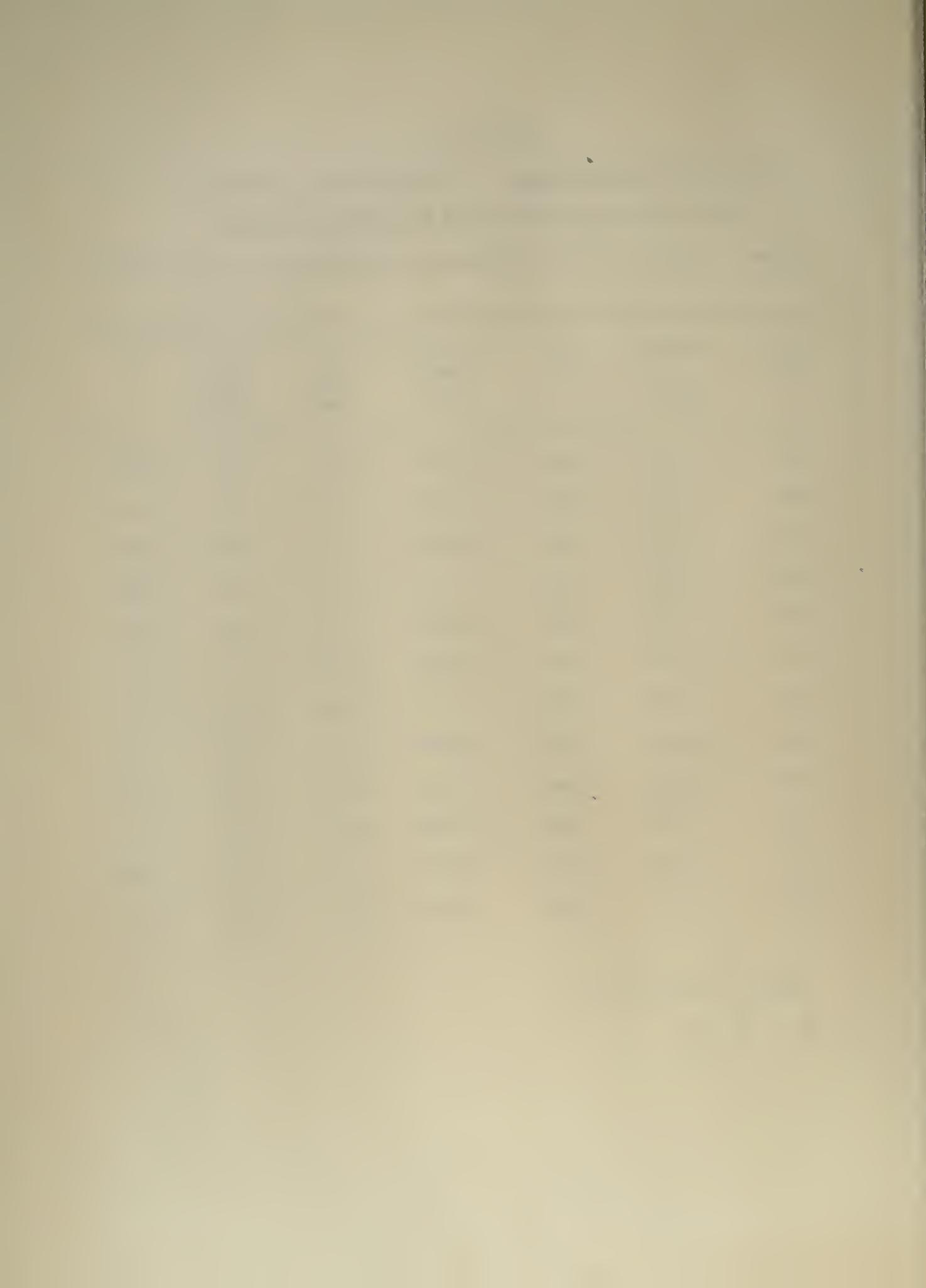
## TABLE VI (cont'd)

## AXIAL COMPRESSION TESTS OF THIN MYLAR CYLINDERS

## WITH FREE PROTRUDING-IN PLEXIGLASS RINGS

Cylinder Radius R = 2.5 in., Mylar Nominal Thickness, $t_1$ = 0.0075 in.

| Test No. | Length, L (in.) | $\frac{L}{R}$ | Thickness, t (in.) | Load, $P_{cr}$ (lb.) | Stress $\sigma_{cr}$ (psi) | $K^*$ |
|------|------|------|--------|------|-----|-------|
| 161 | 0.75 | 0.30 | 0.0075 | 69.3 | 588 | 0.251 |
| 162 | 0.75 | 0.30 | 0.0080 | 72.4 | 576 | 0.246 |
| 163 | 0.75 | 0.30 | 0.0080 | 71.8 | 571 | 0.244 |
| 164 | 0.75 | 0.30 | 0.0080 | 73.6 | 586 | 0.250 |
| 165 | 0.75 | 0.30 | 0.0080 | 73.0 | 581 | 0.248 |
| 166 | 0.75 | 0.30 | 0.0080 | 73.6 | 586 | 0.251 |
| 167 | 0.50 | 0.20 | 0.0076 | 70.6 | 592 | 0.253 |
| 168 | 0.50 | 0.20 | 0.0076 | 71.2 | 596 | 0.255 |
| 169 | 0.50 | 0.20 | 0.0080 | 74.2 | 591 | 0.253 |
| 170 | 0.50 | 0.20 | 0.0080 | 72.4 | 576 | 0.246 |
| 171 | 0.50 | 0.20 | 0.0080 | 74.2 | 591 | 0.253 |

Cylinder Length: 6 in.

$$^*K = (\frac{\sigma_{cr}}{E} \frac{R}{t_1})$$
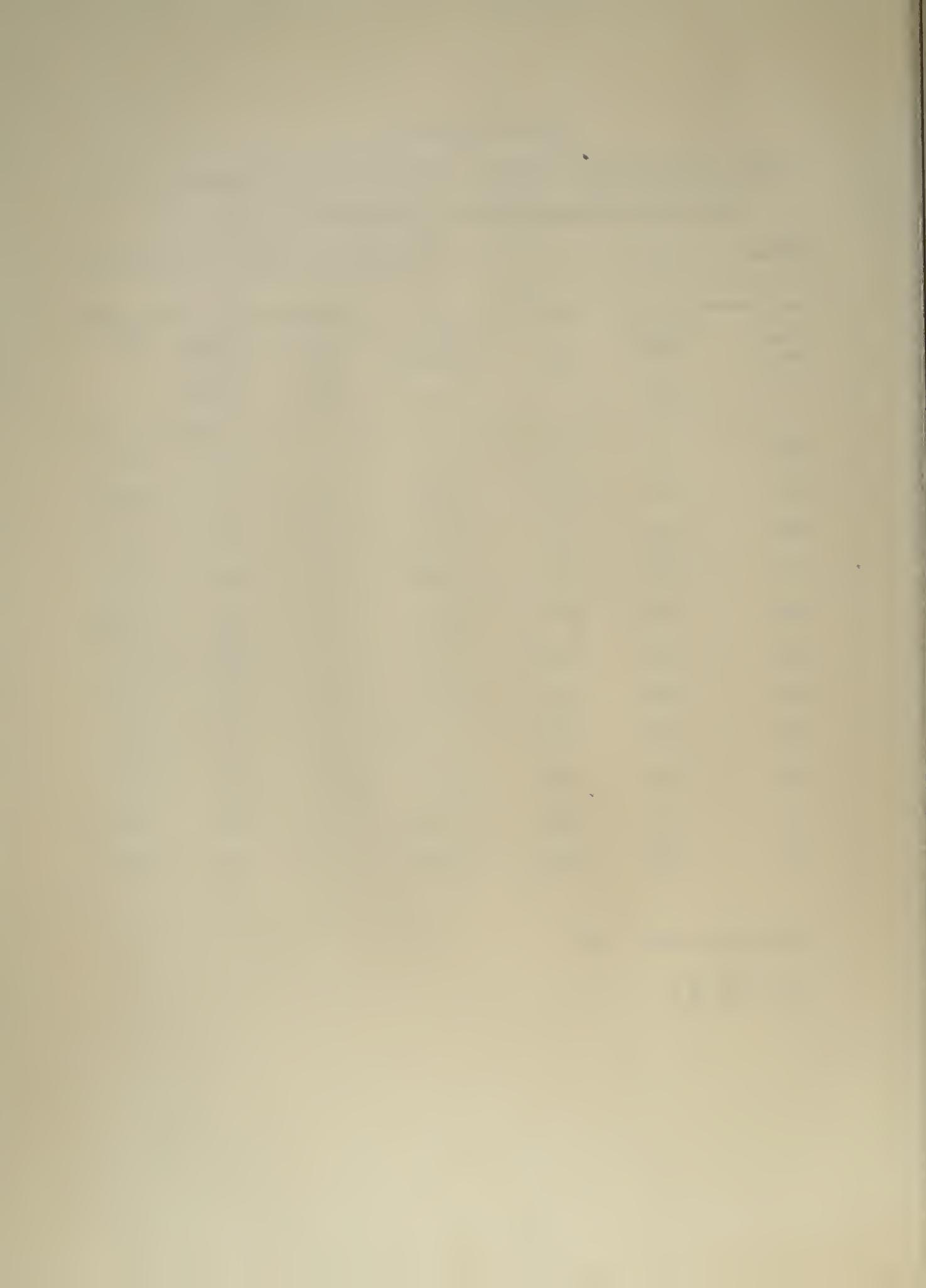
## TABLE VII

### AXIAL COMPRESSION TESTS OF THIN MYLAR CYLINDERS

### WITH FREE PROTRUDING-OUT PLEXIGLASS RINGS

Cylinder Radius R = 2.5 in., Mylar Nominal Thickness, $t_1$ = 0.0075 in.

| Test No. | Length, L (in.) | $\frac{L}{R}$ | Thick-ness, t (in.) | Load, $P_{cr}$ (lb.) | Stress $\sigma_{cr}$ (psi) | $K^*$ |
|------|------|------|------|------|------|------|
| 172 | 2.50 | 1.00 | 0.0070 | 69.3 | 630 | 0.269 |
| 173 | 0.875 | 0.35 | 0.0072 | 79.5 | 704 | 0.301 |
| 174 | 0.875 | 0.35 | 0.0072 | 80.8 | 715 | 0.306 |
| 175 | 0.75 | 0.30 | 0.0066 | 78.4 | 756 | 0.323 |
| 176 | 0.75 | 0.30 | 0.0070 | 87.4 | 795 | 0.340 |
| 177 | 0.50 | 0.20 | 0.0068 | 87.4 | 817 | 0.349 |

Cylinder Length: 6 in.

$$^*K = (\frac{\sigma_{cr}}{E} \frac{R}{t_1})$$
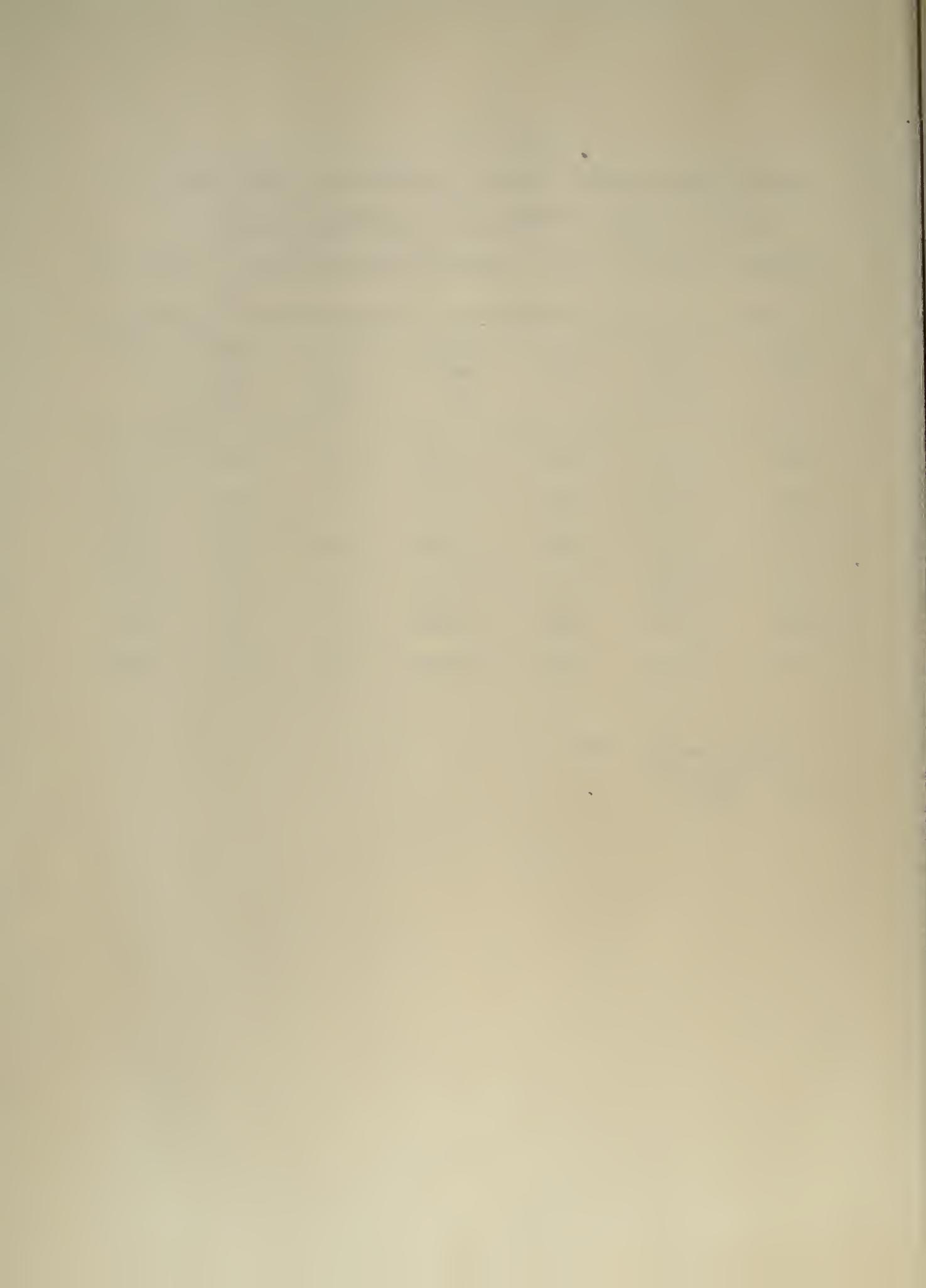
## TABLE VIII

## AXIAL COMPRESSION TESTS OF THIN MYLAR CYLINDERS

## WITH FIXED PROTRUDING-IN PLEXIGLASS RINGS

Cylinder Radius R = 2.5 in., Mylar Nominal Thickness, $t_1$ = 0.0075 in.

| Test No. | Length, L (in.) | $\frac{L}{R}$ | Thickness, t (in.) | Load, $P_{cr}$ (lb.) | Stress $\sigma_{cr}$ (psi) | $K^*$ |
|---|---|---|---|---|---|---|
| 178 | 2.0 | 0.8 | 0.0082 | 84.4 | 655 | 0.280 |
| 179 | 2.0 | 0.8 | 0.0082 | 85.6 | 665 | 0.284 |
| 180 | 2.0 | 0.8 | 0.0082 | 83.7 | 651 | 0.278 |
| 181 | 1.5 | 0.6 | 0.0076 | 76.0 | 636 | 0.272 |
| 182 | 1.5 | 0.6 | 0.0076 | 76.0 | 636 | 0.272 |
| 183 | 1.0 | 0.4 | 0.0076 | 78.4 | 656 | 0.281 |
| 184 | 1.0 | 0.4 | 0.0076 | 80.8 | 676 | 0.289 |
| 185 | 1.0 | 0.4 | 0.0076 | 79.0 | 661 | 0.283 |
| 186 | 0.75 | 0.3 | 0.0076 | 82.6 | 693 | 0.296 |
| 187 | 0.75 | 0.3 | 0.0076 | 82.6 | 693 | 0.296 |
| 188 | 0.50 | 0.2 | 0.0078 | 112.1 | 915 | 0.391 |

Cylinder Length: 6 in.

$$^*K = (\frac{\sigma_{cr}}{E} \frac{R}{t_1})$$
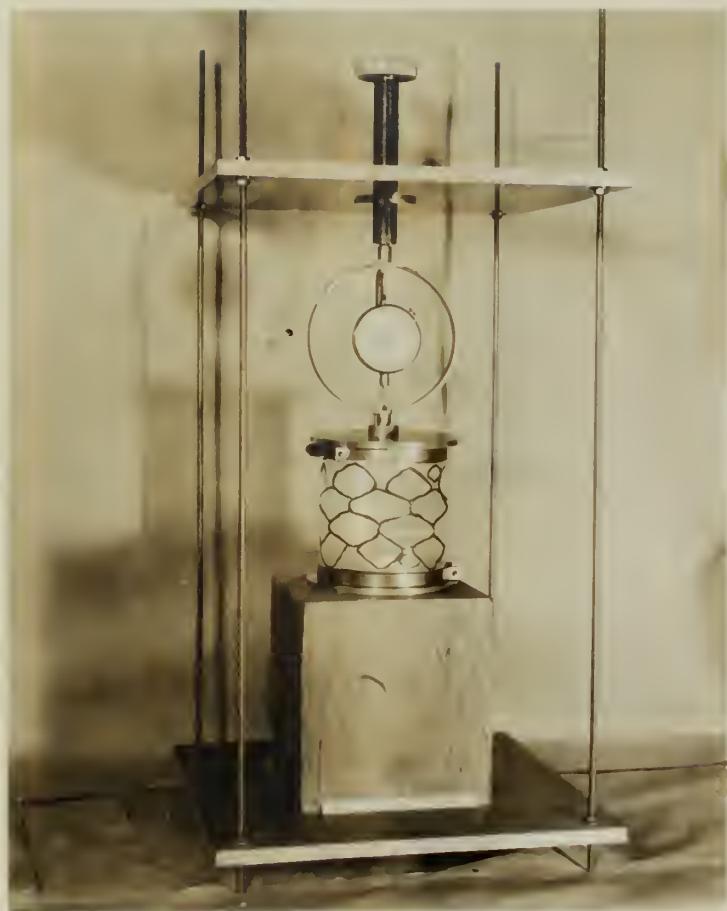
Fig. 1    Photograph of Cylinder Loading Apparatus with Buckled
Mylar Cylinder with 1-inch Spaced Fixed, Protruding-in
Plexiglass Ring-Stiffeners.

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Fig. 2    Photograph of Buckled Mylar Cylinder with Rigidly
Clamped Ends.

Fig. 3    Photograph of Buckled Mylar Cylinder with 1/2-inch
Spaced Thin Mylar-Scotchtape Ring-Stiffeners.

Fig. 4     Photograph of Buckled Mylar Cylinder with 1-inch
Spaced Thick Mylar-Scotchtape Ring-Stiffeners.

Fig. 5    Photograph of Buckled Mylar Cylinder with 7/8-inch
          Spaced Fixed, Protruding-out Plexiglass Ring-Stiffeners.

Fig. 6     Photograph of Buckled Mylar Cylinder with 5/8-inch
Spaced Fixed, Flat Unbroken Plexiglass Ring-Stiffeners.

Fig. 7     Photograph of Buckled Mylar Cylinder with 1-1/2-inch
           Spaced Free, Protruding-in Plexiglass Ring-Stiffeners.

Fig. 8    Photograph of Buckled Mylar Cylinder with 2-1/2-inch
Spaced Free, Protruding-out Plexiglass Ring-Stiffeners.
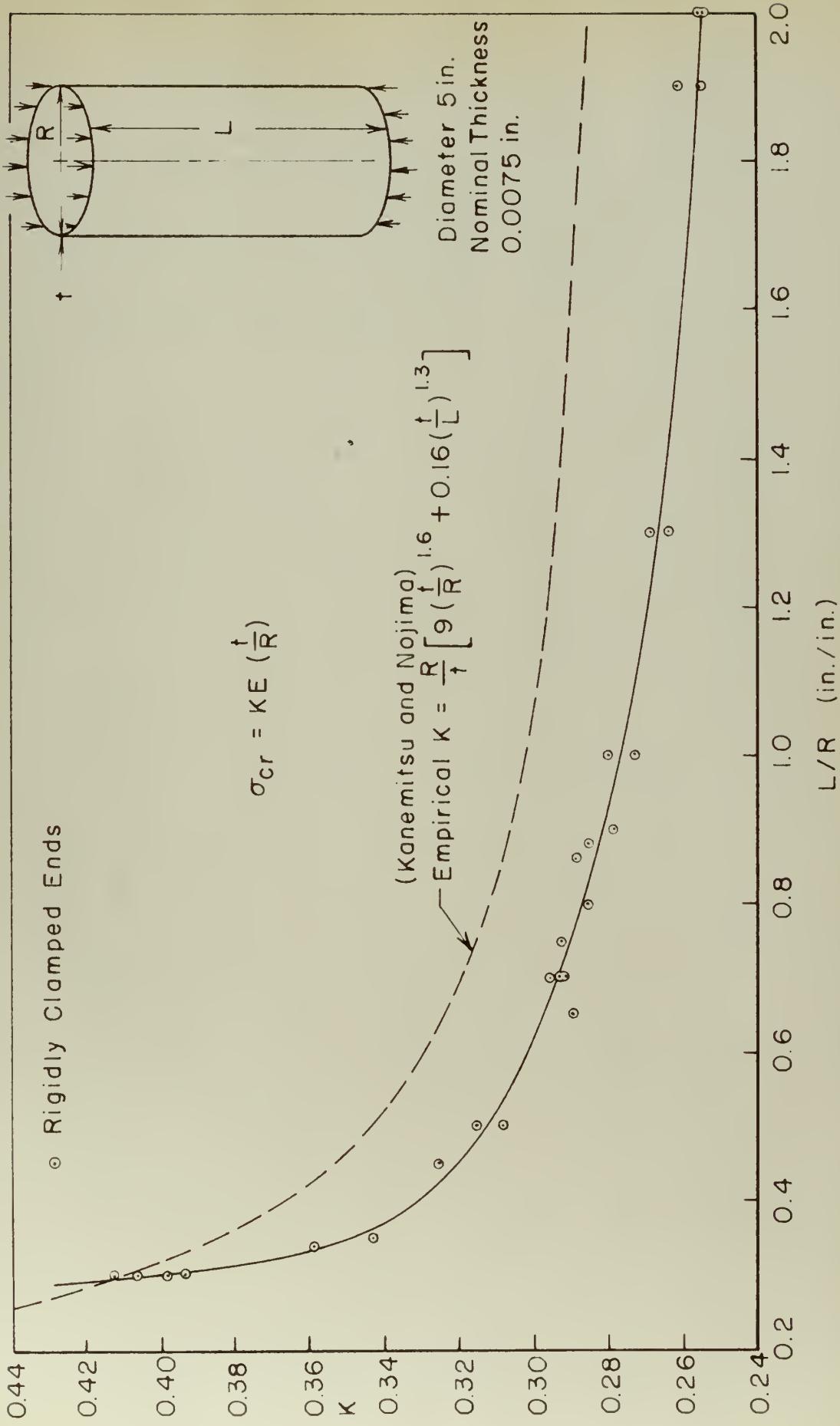
FIG. 9 – AXIAL COMPRESSION TEST OF THIN-WALLED MYLAR CYLINDERS WITH RIGIDLY CLAMPED ENDS

⊙ Rigidly Clamped Ends

Diameter 5 in.
Nominal Thickness 0.0075 in.

$$\sigma_{cr} = KE \left( \frac{t}{R} \right)$$

(Kanemitsu and Nojima)
Empirical $K = \dfrac{R}{t} \left[ 9 \left( \dfrac{t}{R} \right)^{1.6} + 0.16 \left( \dfrac{t}{L} \right)^{1.3} \right]$

L/R (in./in.)

K
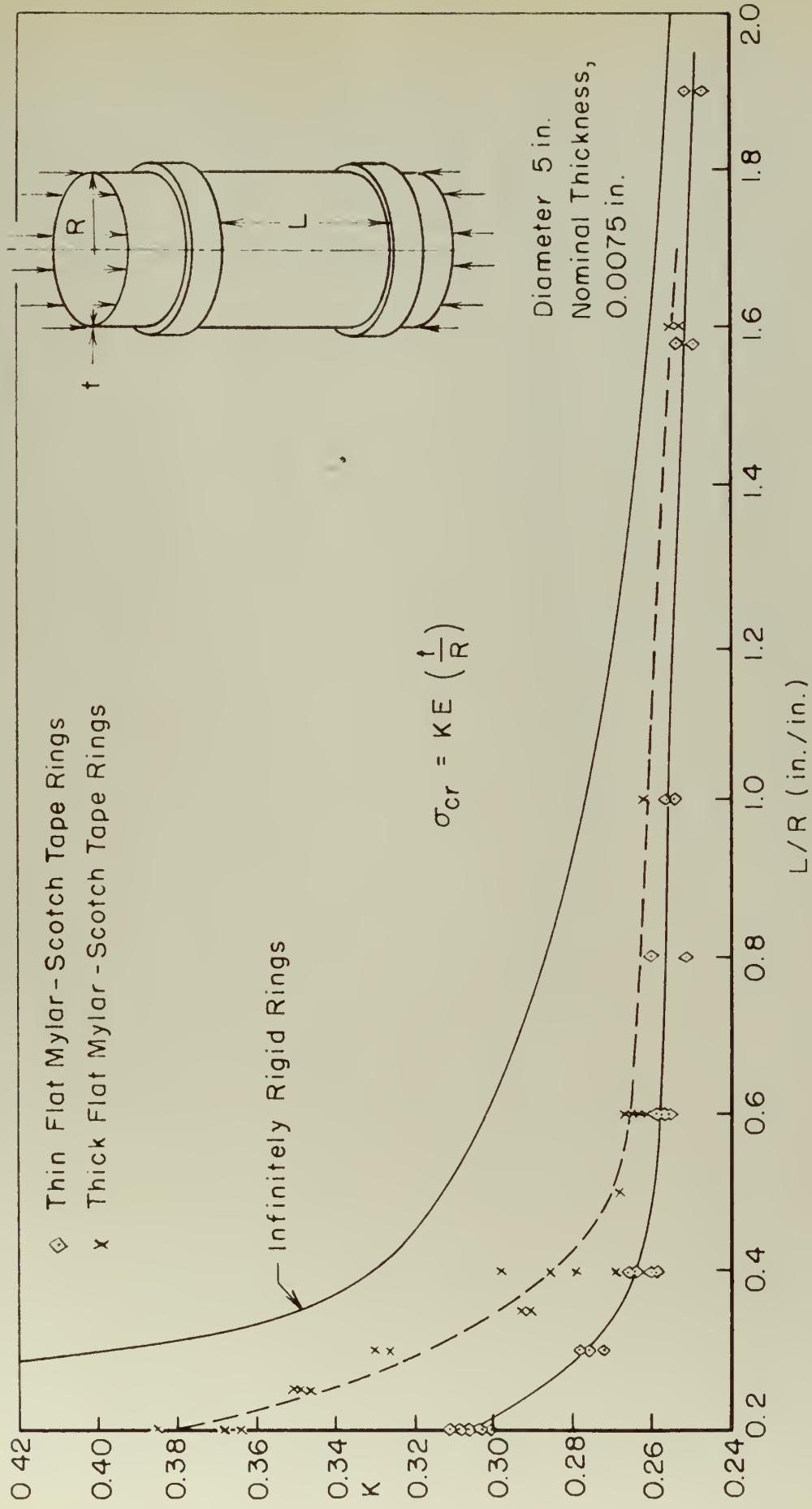
FIG. IO – AXIAL COMPRESSION TEST OF THIN–WALLED MYLAR CYLINDERS WITH MYLAR–SCOTCH TAPE RING STIFFENERS
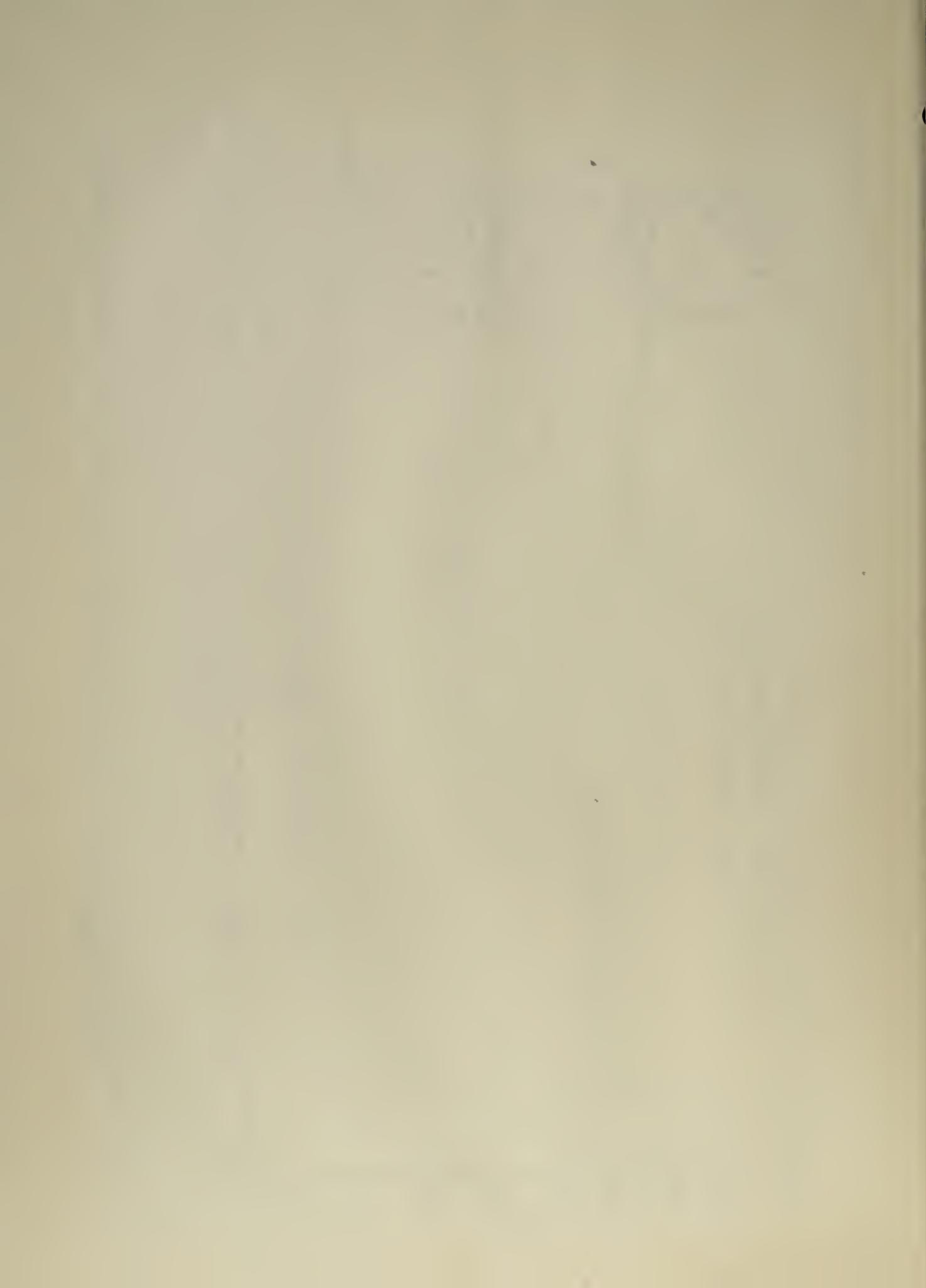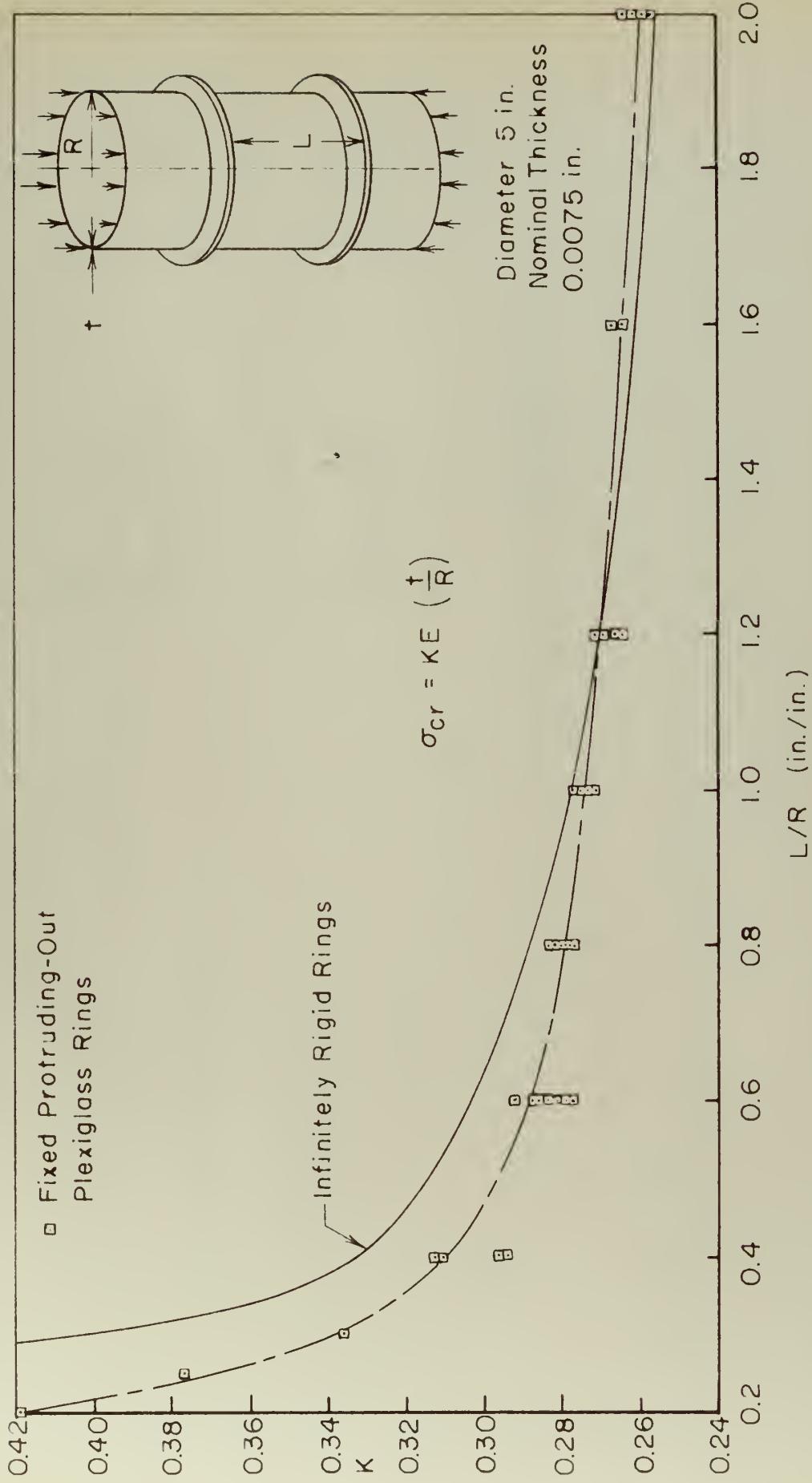
FIG. II—AXIAL COMPRESSION TEST OF THIN-WALLED MYLAR CYLINDERS WITH PLEXIGLASS RING STIFFENERS

FIG.12 – AXIAL COMPRESSION TEST OF THIN-WALLED MYLAR CYLINDERS WITH PLEXIGLASS RING STIFFENERS

Diameter 5 in.
Nominal Thickness 0.0075 in.

$\sigma_{cr} = KE \left(\dfrac{t}{R}\right)$

⊗  Fixed Flat Unbroken Plexiglass Rings
●  Fixed Flat Split Plexiglass Rings
▲  Free Protruding–In Plexiglass Rings

Infinitely Rigid Rings
Fixed Protruding–Out Plexiglass Rings

K

L/R (in./in.)
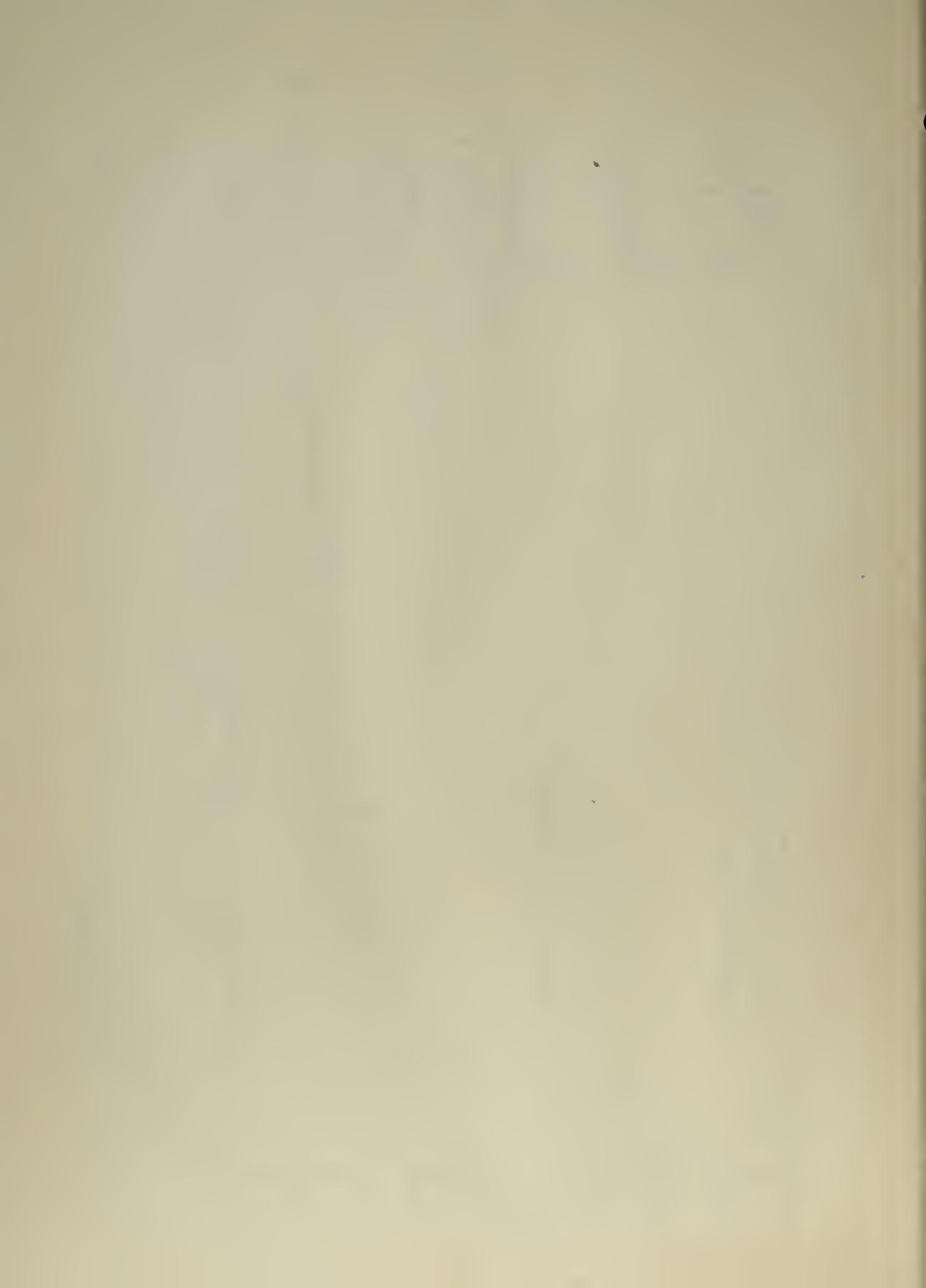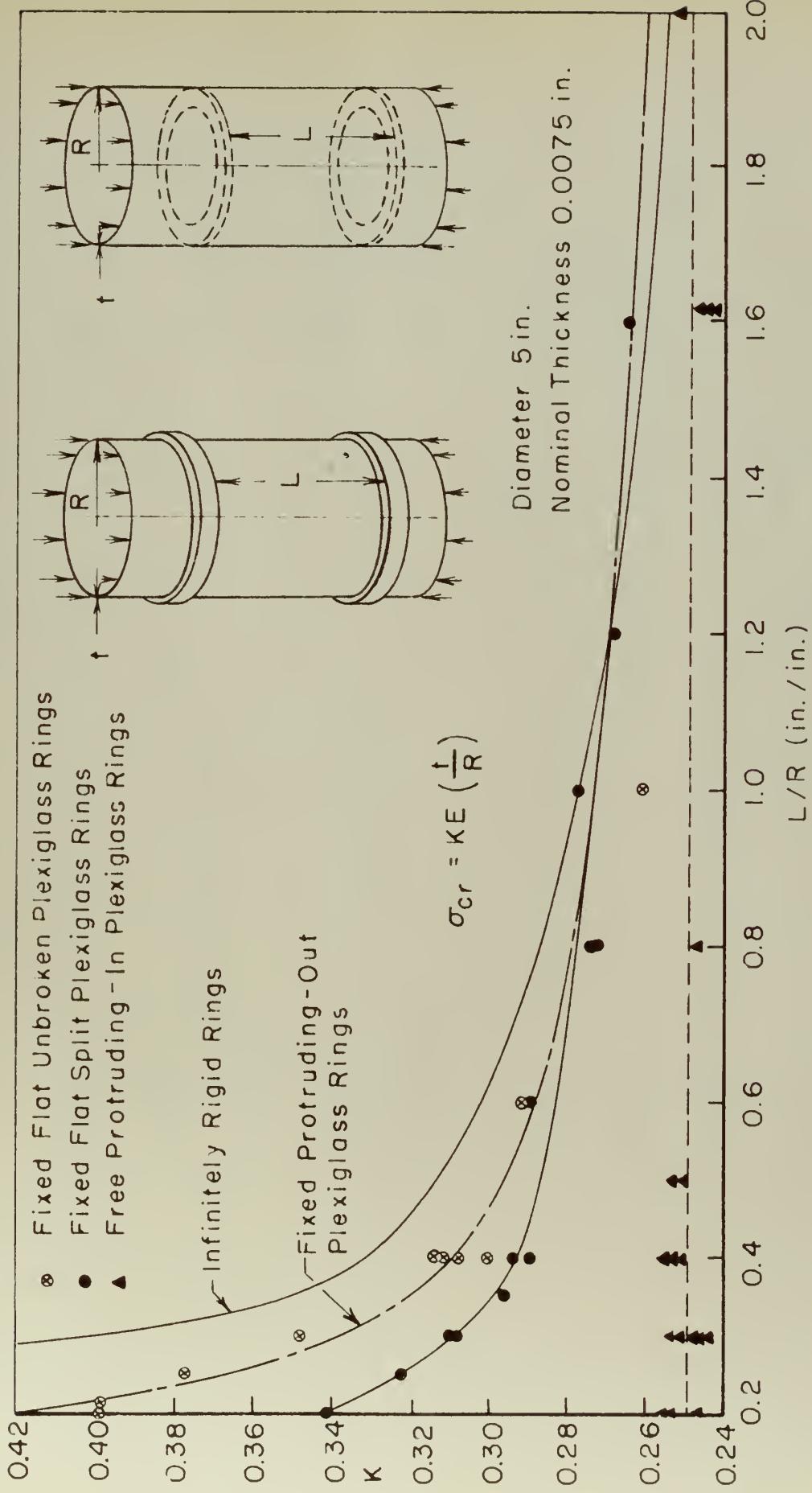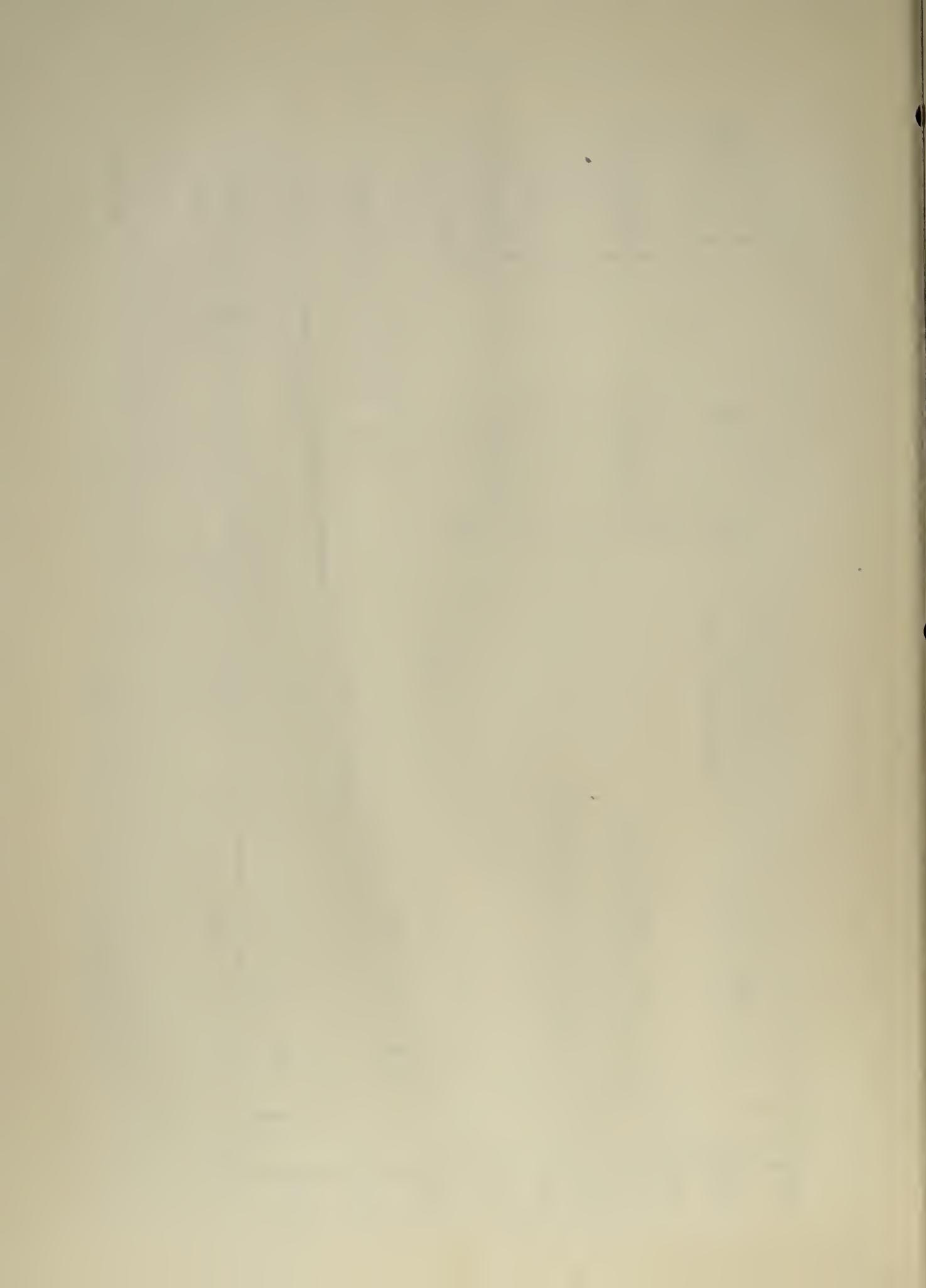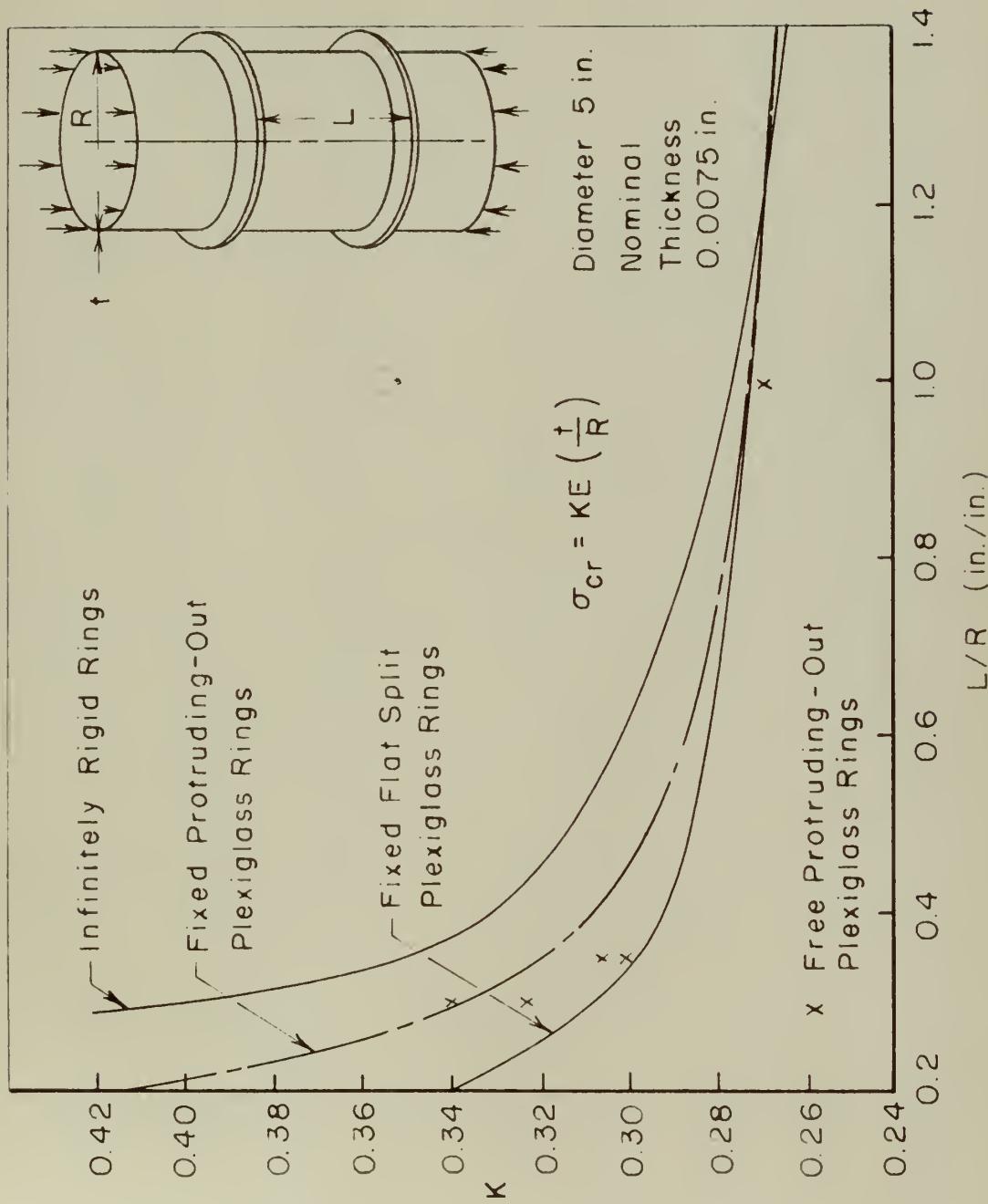
FIG. 13 – AXIAL COMPRESSION TEST OF THIN – WALLED MYLAR CYLINDERS WITH PLEXIGLASS RING STIFFENERS

FIG 14 - AXIAL COMPRESSION TEST OF THIN-WALLED MYLAR CYLINDERS WITH PLEXIGLASS RING STIFFENERS